Turbo-coded Hybrid ARQ for DS-CDMA with Frequency-domain Equalization

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Abstract In this paper hybrid ARQ is applied to DS-CDMA with minimum mean square error frequency-domain equalization (MMSE-FDE). The throughput of turbo-coded HARQ is evaluated in addition to the packet error rate (PER). DS-CDMA with MMSE-FDE can provide much better PER performance than with rake combining in a frequency-selective fading channel and the performance improves with the increase in the frequency-selectivity of the channel. Hence, the throughput is also higher than that with rake combining for all channel conditions.

Keyword mobile communications, DS-CDMA, hybrid ARQ, frequency-selective channel

1. Introduction

In broadband mobile radio communications, the transmitted signal is scattered by many obstacles located between a transmitter and a receiver, thereby creating a propagation channel with numerous paths having different time delays. The transfer function of such a broadband channel is no more constant over the signal bandwidth and is referred to as the frequency-selective fading channel [1]. For successful communications in such fading channels, some powerful multi-access schemes and error control techniques are necessary.

Wideband direct sequence code division multiple access (DS-CDMA) has been adopted in the 3rd generation mobile communication (3G) systems [2]. Recently, a new CDMA system based on the combination of CDMA and multicarrier (MC) modulation based on orthogonal frequency division multiplexing (OFDM), called MC-CDMA, has been attracting much attention [3] and is under extensive study. In [4] it is shown that MC-CDMA with minimum mean square frequency domain equalization (MMSE-FDE) provides a higher forward link capacity than DS-CDMA with rake combining. Lately, the use of FDE in multicode DS-CDMA as in MC-CDMA was proposed [5, 6] and it was shown that multicode DS-CDMA with MMSE-FDE gives much better bit error rate (BER) performance than that with rake combining in a frequency-selective fading channel and is comparable to that of MC-CDMA. In DS-CDMA with MMSE-FDE, the BER performance improves with the increase in the number of propagation paths (or with the increase in the frequency-selectivity) owing to the larger frequency-diversity gain.

Broadband wireless packet technology is one of the core technologies for the next generation mobile communications systems. For packet transmissions, the frequency-selectivity of the channel is not always desirable. With higher frequency-selectivity, the errors are randomized, however, for packet transmissions burst errors are preferable to random errors. Hence there is a need to evaluate how the packet error rate (PER) of a multicode DS-CDMA with MMSE-FDE would be affected in a frequency-selective channel. For packet transmission, some form of error control technique is necessary. Turbo-coded hybrid ARQ (HARQ) is
of the first block of \( N_c + N_g \) chips for the explanation purpose of FFT and IFFT operations performed at the receiver. The resultant multicode DS-CDMA signal before GI insertion may be expressed using the equivalent lowpass representation as

\[
s(t) = \sum_{c=0}^{C-1} 2P/(C \cdot SF) x_c \left[ t/SF \right] k_{cor,c}(t \mod SF) c_{cor}(t) \tag{1}
\]

for \( t = 0 \sim N_c - 1 \), where \( P \) represents the total transmit power. The resultant GI-inserted multicode DC-CDMA signal \( \tilde{s}(t); t = -N_g \sim N_c - 1 \) is transmitted over the frequency-selective channel, where

\[
\tilde{s}(t) = s(t \mod N_c) . \tag{2}
\]

The channel is assumed to be composed of \( L \) distinct propagation paths with different time delays. Block fading, where the path gains stay constant over one block, is assumed. \( M \)-antenna diversity reception is considered. The complex path gain and time delay of the \( l \)th path corresponding to the \( m \)th antenna are respectively denoted by \( \xi_{m,l} \) and \( \tau_l \). The multicode DS-CDMA signal received on the \( m \)th antenna is sampled at the chip rate to obtain \( \tilde{r}_m(t); t = -N_g \sim N_c - 1 \) given as

\[
\tilde{r}_m(t) = \sum_{l=0}^{L-1} \xi_{m,l} \tilde{s}(t - \tau_l) + \eta_m(t) . \tag{3}
\]

where \( \{ \eta_m(t) \} \) is the zero-mean Gaussian process with variance \( 2N_0T_c \) due to the additive white Gaussian noise (AWGN) having the one-sided power spectrum density \( N_0 \). Ideal sampling timing is assumed.

Without loss of generality, we assume the transmission

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At the receiver, $N_c$-point FFT is applied to each block of $N_c$ chips after GI removal to decompose the received multicode DS-CDMA signal into the $N_c$-frequency components. The $k$th frequency component is

$$R_m(k) = \sum_{i=0}^{N_c-1} r_m(t) \exp(-j2\pi k t / N_c)$$

$$= H_m(k) S(k) + \tilde{\eta}_m(k)$$

for $k=0$ to $N_c-1$, where $H_m(k)$ and $S(k)$ are the Fourier transforms of the channel impulse response and the transmitted multicode DS-CDMA signal waveform, respectively, and are given by

$$H_m(k) = \sum_{l=0}^{L-1} h_{ml} \exp(-j2\pi l k / SF)$$

$$S(k) = \sum_{l=0}^{L-1} s(l) \exp(-j2\pi l k / SF)$$

In Eq.(4), $\tilde{\eta}_m(k)$ represents the noise component at the $k$th frequency component.

$R_m(k)$ is multiplied by the weight $w_m(k)$ for joint FDE and antenna diversity combining to obtain

$$\tilde{R}(k) = \sum_{m=0}^{M-1} R_m(k) w_m(k).$$

The weight $w_m(k)$ used in MMSE-FDE for $R_m(k)$ is given by

$$w_m(k) = \frac{H_m^*(k)}{\sum_{m=0}^{M-1} |H_m(k)|^2 + \left[ \frac{C}{SF} \left( 1 + \frac{N_c}{N_p} \right) \right]^{-1} \left( \frac{E_s}{N_0} \right)}^{-1}.$$  (7)

where $E_s/N_0$ represents the average received signal energy per symbol-to-AWGN power spectrum density ratio per antenna and $(.)^*$ denotes the complex conjugate operation. The factor of $(1+4/N_p)$ reflects the power penalty due to GI insertion.

After MMSE-FDE, IFFT is carried out to get back the time-domain spread signal:

$$\tilde{s}(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{R}(k) \exp(j2\pi k t / N_c)$$

for $t=0$ to $N_c-1$. After IFFT, despreading is carried out to obtain

$$\hat{x}(i) = \sum_{i=0}^{(i+1)SF-1} \tilde{s}(i) \delta(i)(modSF)c_{sc}(t)$$

for $c=0$ to $C-1$ and $i=0$ to $N_c/FS-1$. The soft values $\{\hat{x}(i)\}$ are parallel-to-serial (P/S) converted and soft QPSK demodulated to recover the soft decision sample sequence.

After the entire block of $K$ symbols is received, deinterleaving and turbo decoding are performed. NAK (or ACK) signal is transmitted via reverse channel to the transmitter side in case packet error (or no packet error) is detected. Two types of HARQ schemes are considered. The first is the type I HARQ in which the parity bits are transmitted together with the systematic (information) bits in each packet [9]. For retransmitted packets, time diversity combining (TDC) [10] is used. The type II HARQ is also considered wherein only the systematic bits are transmitted in the first transmission, and only if a retransmission is requested, the punctured parity bits are transmitted [9]. Incremental redundancy (IR) [11] and TDC (in case the same packet is retransmitted) are utilized.

3. Simulation Results

The PER and throughput performances of multicode DS-CDMA with MMSE-FDE are compared with those of ideal rake combining. For rake combining, GI insertion is not necessary. For the simulation purposes, we assume a frequency-selective Rayleigh fading channel having a 16-path exponential power delay profile with a time-delay spacing of 1 chip and the power difference of $\alpha$ dB between adjacent paths (uniform power delay profile when $\alpha=0$ dB) and a normalized maximum Doppler frequency $f_D T_c N_c$ of 0.001, where $f_D$ is the maximum Doppler frequency given by traveling speed/carryr wavelength and $T_c$ is the chip duration. Block fading, where the path gains stay constant over one block of $N_c$ chips is assumed. In the simulation, $N_c=256$ and $SF=256$ are assumed. The information sequence length is taken to be 1024 bits. Turbo encoding with a coding rate of 1/3 and a constraint length of 4, and log-MAP decoding with 8 iterations are used.

For type I HARQ, two parity sequences obtained after turbo encoding are punctured with a puncturing period of 2 (resulting in a rate ½ turbo code) and transmitted together with the information bit sequence. In case a retransmission is requested, the same packet is transmitted again. For an information bit sequence with $N$ bits, the type I HARQ packet consists of $2N$ bits. For type II HARQ assumed in this paper, each transmitted packet consists of $N$ bits. First transmission consists of systematic bits only. Second transmission consists of the punctured bit sequence obtained by puncturing the two parity sequences with a puncturing period of 2. In case another retransmission is requested, the parity bits, that were not transmitted in the previous transmission, are transmitted. So after 3 transmissions a rate 1/3 turbo code is received. Ideal chip synchronization and ideal channel estimation are assumed at the receiver. Perfect error detection and an error-free reverse channel are assumed.
3.1. Uncoded PER

The computer simulation results are presented for the maximum code-multiplex order, i.e., $C = SF$. Figure 2 plots the uncoded PER for DS-CDMA with MMSE-FDE and rake combining as a function of the signal energy per information bit-to-AWGN power spectral density ratio $E_b/N_0$ with $\alpha$ as a parameter. The BER performance (not plotted) degrades with the increase in the frequency-selectivity of the channel, i.e., increase in $\alpha$ [5]. With the increase in the frequency-selectivity of the channel, the code orthogonality destruction is severer and the performance degrades. From Fig. 2 it can be seen that the PER for rake combing is almost 1 and also improves slightly with the decrease in the frequency-selectivity of the channel. However for DS-CDMA with MMSE-FDE, the effect of the channel’s frequency-selectivity is different. Orthogonality destruction is partially restored by the MMSE equalization and the frequency-diversity gain improves with increasing frequency-selectivity.

![Figure 2. Uncoded PER for MMSE-FDE and rake combining.](image)

3.2. Turbo-coded PER

Figure 3 plots the turbo-coded PER for DS-CDMA with MMSE-FDE and rake combining as a function of $E_b/N_0$ when a rate $\frac{1}{2}$ turbo coding is used with $\alpha$ as a parameter. It is again clearly seen that MMSE-FDE provides better performance than rake combining. When rake combining is used, the PER increases with decreasing $\alpha$ (i.e., increasing frequency-selectivity) since the frequency-selectivity of the channel becomes stronger and destruction of code orthogonality is severer. On the contrary, when MMSE-FDE is applied, the PER decreases with decreasing $\alpha$. This is because MMSE-FDE can restore partially the code orthogonality while achieving a larger frequency-diversity effect in a channel with stronger frequency-selectivity.

![Figure 3. Turbo-coded PER for MMSE-FDE and rake combining.](image)

3.3. Throughput of Type I and Type II HARQ

Throughput in bits per second per Hertz (bps/Hz) is plotted in Figure 4 as a function of the signal energy per transmitted bit-to-AWGN power spectral density ratio $E_t/N_0$ (we use $E_t$ because $E_b$ is dependent on throughput). The maximum attainable throughput is 1bps/Hz with type I HARQ since the parity bits are always present, while it is 2bps/Hz with type II HARQ since no parity bits are transmitted in the first transmission and the parity bits are transmitted only on request. The throughput for rake combining is almost always seen to be lower than that for MMSE-FDE. MMSE-FDE improves the throughput for both type I and type II HARQ, but the improvement is larger for type II HARQ.

For the type I HARQ, when rake combining is used, the throughput is lower for lower $\alpha$. When MMSE-FDE is applied instead of rake combining, however, the throughput of type I HARQ is almost insensitive to $\alpha$ and is 1 bps/Hz for $\alpha > 5$dB. For type II HARQ, when rake combining is used, the throughput is only slightly better than that of type I HARQ. This is because, due to severe orthogonality destruction, only the information bits transmitted with the first transmission cannot provide error free transmission and a retransmission is always requested. However, when MMSE-FDE is used, the code orthogonality is partially restored and frequency-diversity effect is also attained so the throughput increases with the increase in $E_t/N_0$; a throughput close to 2bps/Hz is obtained at $E_t/N_0=20$dB, i.e., the information bits are received correctly almost always in the first transmission.
3.4. Antenna Diversity

Antenna diversity is a well-known technique to improve the transmission performance. The throughput of multicode DS-CDMA with MMSE-FDE and rake combining in the presence of multiple receive antennas are compared in Fig. 5 for type II HARQ when $\alpha=4\text{dB}$. It is seen that for both MMSE-FDE and rake combining, the throughput improves with the increase in the number of antennas. The improvement for DS-CDMA with MMSE-FDE is much larger than the improvement for rake combining. With rake combining, the information bits transmitted in the first transmission is always in error even with multiple receive antennas and the second transmission is requested. It is seen that even with antenna diversity reception, the throughput of MMSE-FDE is better than that of rake combining; the difference widens with the increase in the number of antennas.

4. Conclusion

The PER and throughput performances of multicode DS-CDMA with MMSE-FDE were evaluated and compared with those with rake combining. It was found that when MMSE-FDE is used, the PER improves with the increase in the frequency-selectivity of the channel due to partial code orthogonality restoration and larger frequency-diversity in contrast to rake combining. It was also found that the throughput with MMSE-FDE is better than that with rake combining for both type I and type II HARQ; however the throughput improvement is much larger for type II HARQ. Antenna diversity reception improves the throughput performance and is more effective for DS-CDMA with MMSE-FDE.

References
