Performance Improvement with Space-Time Transmit Diversity using Minimum Mean Square Error Combining Equalization in MC-CDMA

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Abstract In this paper, the space time transmit diversity (STTD) decoding combined with minimum mean square error combining (MMSEC) equalization is presented for MC-CDMA. The STTD decoding weights that minimize the MSE for each subcarrier before despreading are derived. From computer simulation, it is found that the BER performance of STTD with MMSEC equalization using the weights derived in this paper is always better than that with no diversity and provides the same diversity order as 2-antenna receive diversity with MMSEC equalization but with 3dB performance penalty. The uplink BER performance can also be improved with STTD, however the error floor still exists.

Keyword mobile radio, MC-CDMA, MMSEC, space time coding, transmit diversity.

1. Introduction

Recently, the combination of multicarrier (MC) modulation based on orthogonal frequency division multiplexing and code division multiple access (CDMA), called MC-CDMA [1], has gained a lot of attention because of its ability to allow high data rate transmission in a harsh mobile environment and has emerged as the most promising candidate for the forth generation of mobile communication systems [2]. In MC-CDMA, each user’s data-modulated symbol to be transmitted is spread over a number of subcarriers using an orthogonal spreading code defined in the frequency-domain. Since the received signal suffers from frequency selective multipath fading, the orthogonality among different users’ signals is partially lost, producing a large multi-user interference (MUI). However, the orthogonality property can be partially restored while achieving the frequency diversity effect by using the minimum mean square error combining (MMSEC) equalization per subcarrier [3] and hence, a better bit error rate (BER) performance can be achieved.

Multiple antennas can be used to reduce the adverse effect of multipath fading. Receive diversity has been successfully used in practical systems. However, recently, transmit antenna diversity has been gaining much attention since the use of transmit diversity at a base station alleviates the complexity problem of mobile receivers [4]. Space-time transmit diversity (STTD) [5] offers a way to introduce a degree of space diversity without going to the complexity of closed-loop transmit diversity solutions. In MC-CDMA, STTD decoding can be performed in conjunction with MMSEC equalization to further improve the BER performance.

In this paper, we propose, joint STTD decoding and frequency-domain equalization for MC-CDMA. The remainder
of this paper is organized as follows. The transmission system model for MC-CDMA with joint STTD decoding and MMSEC equalization is presented in Section 2. The equalization weights for joint STTD decoding and MMSEC equalization are presented in Section 3. In Section 4, the simulation results for the downlink and uplink cases are presented and discussed. Section 5 concludes the paper.

2. Transmission System Model with Joint STTD and MMSEC Equalization

Figure 1 shows the transmission system model with joint STTD and MMSEC equalization. We consider MC-CDMA having \( N_u \) subcarriers with a carrier spacing of \( 1/T_s \). Without loss of generality, we consider the two MC-CDMA signaling intervals, i.e., \( 0 \leq t < 2T \) with \( T=T_s+T_g \), where \( T_s \) and \( T_g \) are respectively the effective symbol length and the guard interval (GI). Throughout the paper, discrete-time representation of the MC-CDMA signal is used.

2.1. Transmit signal

At the transmitter, each user’s data is spread using the frequency-domain orthogonal spreading code with a spreading factor \( SF \). Let \( d^u(n) \) be the \( n \)th data-modulated symbol with \( |d^u(n)|=1 \). The STTD encoder encodes the modulated symbols \( \{d^u(n); n=0 \sim 2(N_s/SF)-1\} \) for transmission during the signaling interval \( 0 \leq t < 2T \) from the two antennas over the \( N_c \) subcarriers, where \( N_c/SF \) is an integer. The serial-to-parallel (S/P) converter converts the STTD encoded data into \( N_c/SF \) parallel data streams, each of which is copied \( SF \) times and multiplied by the orthogonal spreading code \( \{c^u(k); k=0\sim SF-1\} \) for the \( a \)th user. It is then further multiplied by a long scramble sequence \( \{pn(k); k=\cdots,-1,0,1,\cdots\} \). The STTD encoded data waveform \( y^u_m(n,t) \) to be transmitted from the \( m \)th antenna, \( m=0 \) and 1, over the \( q \cdot SF \) th \( \sim (q+1) \cdot SF-1 \) th subcarriers, \( q=0\sim N_s/SF-1 \), is as shown in Table 1.

Uplink is considered first. The \( a \)th user’s \( k \)th subcarrier component \( \{x^u_m(k,t); k=0 \sim N_c-1\} \), transmitted on the \( m \)th antenna during the time interval \( t=0\sim T \) and \( t=T\sim 2T \), may be expressed using the equivalent baseband representation as

\[
x^u_m(k,t) = \sqrt{A} e^{j(k \text{ mod } SF)} p^n(k) y^u_{m} \left( \frac{k}{SF} \right) t
\]

for uplink, (1)

where \( A \) represents the total transmit power of each user, \( y^u_{m}(n,t) \) is as shown in Table 1 for the respective signaling interval and antenna, and \( \lfloor a \rfloor \) denotes the largest integer smaller than or equal to \( a \). In Table 1 and henceforth, \( (\cdot) \) denotes the complex conjugate operation. \( N_c \)-point inverse fast Fourier transform (IFFT) is applied to the sequence \( \{x^u_m(k,t); k=0 \sim N_c-1\} \) to generate the MC-CDMA signal \( \{s^u_m(i,t); i=0 \sim N_c-1\} \):

\[
s^u_m(i,t) = \sum_{k=0}^{N_c-1} x^u_m(k,t) \exp \left( j2\pi k \frac{i}{N_c} \right).
\]

where \( i \) represents the sample position in the signaling intervals \( t=0\sim T \) and \( t=T\sim 2T \). After insertion of the \( N_g \) sample GI, the resultant MC-CDMA signal \( \{z^u_m(i,t); i=-N_g \sim N_c-1\} \) is transmitted over a propagation channel, where

\[
z^u_m(i,t) = s^u_m(i \text{ mod } N_c, t).
\]

In the downlink case, the \( U \) users’ symbols spread on each subcarrier are added and then multiplied by the long scramble sequence. Hence the resultant \( k \)th subcarrier component \( x^u_n(k,t) \) is given by

\[
x^u_n(k,t) = \sqrt{A} \sum_{m=0}^{U-1} c^u(k \text{ mod } SF) p^n(k) y^u_{m} \left( \frac{k}{SF} \right) t
\]

for downlink. (4)

The resulting MC-CDMA signal \( \{x^u_n(i,t); i=0 \sim N_c-1\} \) and the GI-inserted MC-CDMA signal \( \{z^u_n(i,t); i=-N_g \sim N_c-1\} \) are given by Eqs. (2) and (3), respectively, with \( \{x^u_n(k,t)\} \) replaced by \( \{x^u_n(k,t)\} \), i.e., omitting the superscript \( u \) representing the \( a \)th user.

The IFFT sampling period is taken to be \( \Delta T=T/N_c \), such that \( T=NT_s \Delta T \) and \( T=T_s+T_g=T_s(1+N_g/N_u) \).

Table 1: STTD encoded data waveform \( y^u_{m}(n,t) \) transmitted over the \( q \cdot SF \) th \( \sim (q+1) \cdot SF-1 \) th subcarriers.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( q=0 )</th>
<th>( q=1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t=0\sim T )</td>
<td>( d^u= d^u(2q) )</td>
<td>( d^u= d^u(2q+1) )</td>
</tr>
<tr>
<td>( t=T\sim 2T )</td>
<td>( -d^u= -d^u(2q+1) )</td>
<td>( d^u= d^u(2q) )</td>
</tr>
</tbody>
</table>

2.2. Channel Model

Let independent propagation paths with distinct time delays \( \{\tau^m_i\} \) is assumed. For the uplink case, the discrete time impulse response \( \xi^u_m(n) \) of the multipath channel between the \( m \)th transmit antenna and the receive antenna for the \( a \)th user may be expressed as

\[
\xi^u_m(n) = \sum_{i=0}^{L-1} \delta(n-i) \delta(t - \tau^m_i)
\]

(5)

with \( \sum_{i=0}^{L-1} E[\xi^u_m(i)] = 1 \), where \( \delta(t) \) is the delta function and \( E[\cdot] \) denotes ensemble average. It is assumed that the channel impulse response remains the same for the two signaling intervals \( t=0\sim T \) and \( t=T\sim 2T \). The time delays \( \{\tau^m_i\} \) are assumed to be multiples of the FFT sampling period \( \Delta T \).

For the downlink case, all users’ signals go through the same
channel and therefore, the superscript $u$ representing the $u$th user is omitted from Eq. (5).

### 2.3. Received signal

Uplink is considered first. The received MC-CDMA signal is sampled at the rate of $\Delta T = N_s/T_s$ to obtain \( \{\tilde{r}(i,t); i = -N_s \sim N_s - 1\} \), which is expressed as

\[
\tilde{r}(i,t) = \sum_{u=0}^{U-1} \sum_{l=0}^{L-1} \xi_u^a \xi_l^a (i - \tau_i^u) / \Delta T + \sum_{i=0}^{L-1} \sum_{l=0}^{L-1} \xi_l^a \xi_l^b (i - \tau_i^u) / \Delta T + \eta(i,t)
\]

where \( \eta(i,t) \) represents the additive white Gaussian noise (AWGN) process at the sampling instant \( i \) within the signaling interval \( t \) and has zero mean and variance \( 2N_0/T_s \) with \( N_0 \) representing the single sided AWGN power spectrum density. It is assumed for simplicity that even for uplink, all the users are synchronized, but the impact of transmit timing asynchronism is evaluated by computer simulations in Sect. 4. On the other hand, the received MC-CDMA signal for the downlink can be given by Eq. (6) with \( \xi_u^a \) and \( \{\tau_i^u\} \) replaced by \( \xi_m \) and \( \{\tau_i\} \), respectively.

The \( N_s \)-sample GI is removed and the \( N_s \)-point FFT is applied to decompose the received MC-CDMA signal into the \( N_s \)-subcarrier components \( \{r(k,t); k = 0 \sim N_s - 1\} \):

\[
r(k,t) = \sum_{i=0}^{N_s-1} \tilde{r}(i,t) \exp(-j2\pi k i / N_s).
\]

Denoting the \( u \)th user’s channel gain at the \( k \)th subcarrier for the \( m \)th transmit antenna by \( H_m^u(k) \), the \( k \)th subcarrier component \( r(k,0) \) received in the signaling interval \( t=0-T \) may be represented as

\[
r(k,0) = \sum_{u=0}^{U-1} H_m^u(k) x_u^a(k,0) + \sum_{u=0}^{U-1} H_m^u(k) x_u^b(k,0) + \eta(k,0)
\]

for uplink. (8a)

where \( x_u^a(k,t) \) is as defined in Eq. (1). Likewise, the signal received during the signaling interval \( t=T\sim2T \) can be expressed as

\[
r(k,T) = \sum_{u=0}^{U-1} H_m^u(k) x_u^a(k,T) + \sum_{u=0}^{U-1} H_m^u(k) x_u^b(k,T) + \eta(k,T)
\]

for uplink. (8b)

In Eqs. (8a) and (8b), \( \{H_m^u(k); k = 0 \sim N_s - 1\} \) and \( \{\eta(k,t); k = 0 \sim N_s - 1\} \) are respectively the fast Fourier transforms of the channel impulse response \( \xi_m^u(\tau) \) and the AWGN process \( \eta(i,t) \). They are given by

\[
\begin{align*}
H_m^u(k) &= \sum_{l=0}^{L-1} \xi_m^u \exp(-j2\pi l k / N_s) \\
\eta(k,t) &= \sum_{i=0}^{N_s-1} \eta(i,t) \exp(-j2\pi i / N_s)
\end{align*}
\]

(9)

For the downlink, the \( k \)th subcarrier components \( r(k,0) \) and \( r(k,T) \) received in the signaling intervals \( t=0\sim T \) and \( t=T\sim2T \) may be represented as

![Figure 1. Transmission system model.](image-url)
\[
\begin{align*}
    r(k,0) &= H_o(k)x_o(k,0) + H_1(k)x_i(k,0) + \eta(k,0) \\
    \text{for downlink (10a)}
\end{align*}
\]
and
\[
\begin{align*}
    r(k,T) &= H_o(k)x_o(k,T) + H_1(k)x_i(k,T) + \eta(k,T) \\
    \text{for downlink (10b)}
\end{align*}
\]
where \( H_m(k) \) is the channel gain of the downlink at the \( k \)th subcarrier for the \( m \)th transmit antenna and \( x_m(k,t) \) is as defined in Eq. (4).

The STTD decoding combined with MMSEC equalization is carried out using the equalization weights as derived in Section 3 and the resulting soft samples \( \{\tilde{x}(k,t); k = qSF \sim (q+1)SF - 1\} \) are despread, i.e., multiplied by the scramble sequence \( \{pn(k)\} \) and the orthogonal spreading code \( e^w(k \mod SF); k = qSF \sim (q+1)SF - 1\), and summed to obtain the decision variable \( \tilde{d}^w(q,t) \) for the \( q \)th data-modulated symbol of the \( u \)th user transmitted in the signaling interval \( t \) given as
\[
\begin{align*}
    \tilde{d}^w(q,t) &= \sum_{k=qSF}^{(q+1)SF-1} \tilde{x}(k,t) e^w(k \mod SF) p_n(k). \quad (11)
\end{align*}
\]
The recovered modulated symbols for the two signaling intervals \( t=0-T \) and \( t=T-2T \) are then aligned to obtain the \( m \)th data-modulated symbol \( \hat{d}^w(n) \) as
\[
\begin{align*}
    \hat{d}^w(n) &= \begin{cases} 
        \tilde{d}^w(q,0), & \text{if } n = 2q \\
        \tilde{d}^w(q,T), & \text{if } n = 2q + 1
    \end{cases} \quad \text{for } n=0 \sim 2(N/SF)-1. \quad (12)
\end{align*}
\]

3. Equalization Weights for Joint STTD Decoding and MMSEC Equalization

The STTD decoding is performed on each subcarrier and then the frequency-domain despreading is carried out using MMSEC. In this paper, we derive the equalization weights for joint STTD decoding and MMSEC equalization. The STTD decoding is carried out as follows:
\[
\begin{align*}
    \tilde{x}(k,0) &= w_0(k)r(k,0) + w_1(k)r'(k,T) \\
    \tilde{x}(k,T) &= w_1(k)r(k,0) - w_0(k)r'(k,T), \quad (13)
\end{align*}
\]
where \( w_0(k) \) and \( w_1(k) \) are the STTD decoding weights. The decoding weights for both signaling intervals \( t=0-T \) and \( t=T-2T \) are the same since we are assuming that the channel impulse response remains the same over the two signaling intervals; hence we use the first signaling interval to derive the equalization weights. Without loss of generality, we assume that the 0th user is the desired user. The instantaneous error \( \xi(k) \) of the \( k \)th subcarrier soft sample in the signaling interval \( t=0-T \) is defined as
\[
\begin{align*}
    \xi(k) &= \tilde{x}(k,0) - x(k,0), \quad (14)
\end{align*}
\]
where
\[
\begin{align*}
    x(k,0) &= \begin{cases} 
        \frac{1}{\sqrt{SF}} \sum_{n=0}^{A-1} e^n(k mod SF) p_n(k) d_0^n \left( \frac{k}{SF} \right), & \text{for downlink (15)} \\
        \frac{1}{\sqrt{SF}} \sum_{n=0}^{A-1} e^n(k mod SF) p_n(k) d_0^n \left( \frac{k}{SF} \right), & \text{for uplink}
    \end{cases}
\end{align*}
\]
is the symbol transmitted during the signaling interval \( t=0-T \) from the 0th antenna \((m=0)\). We want to find the set of equalization weights \( w_0(k) \) and \( w_1(k) \) that minimizes the mean square error (MSE) \( E[|\xi(k)|^2] \). It is assumed that the \( U \) data-modulated symbols \( \{d^u(n); u = 0 \sim U - 1\} \) are zero-mean and independent random variables. For the downlink, the channel gain for all users’ transmitted symbols are the same. For the uplink, on the other hand, the channel gain is different for different users, i.e., \( H_o^u(k) \neq H_o^{u'}(k) \) and \( H_i^u(k) \neq H_i^{u'}(k) \) if \( u \neq u' \). Slow transmit power control is assumed so that all users’ signals are received at the base station with the same average power.

The set of equalization weights that satisfy \( \frac{\partial E[|\xi(k)|^2]}{\partial w_0(k)} = 0 \) and \( \frac{\partial E[|\xi(k)|^2]}{\partial w_1(k)} = 0 \) is found for downlink and uplink as:
\[
\begin{align*}
    w_0(k) &= \begin{cases} 
        H_o(k), & \text{for downlink (16)} \\
        H_o^u(k), & \text{for uplink (17)}
    \end{cases}
\end{align*}
\]
where
\[
\begin{align*}
    w_0(k) &= \begin{cases} 
        \left[ H_o(k) \right]^2 + \left[ H_1(k) \right]^2 + \left( \frac{U}{SF} \frac{AT_o}{2N_0} \right), & \text{for downlink (16)} \\
        \left[ H_o^u(k) \right]^2 + \left[ H_i^u(k) \right]^2 + \left( \frac{1}{SF} \frac{AT_o}{2N_0} \right), & \text{for uplink (17)}
    \end{cases}
\end{align*}
\]
and
\[
\begin{align*}
    w_1(k) &= \begin{cases} 
        \left[ H_o(k) \right]^2 + \left[ H_1(k) \right]^2 + \left( \frac{U}{SF} \frac{AT_o}{2N_0} \right), & \text{for downlink (16)} \\
        \left[ H_o^u(k) \right]^2 + \left[ H_i^u(k) \right]^2 + \left( \frac{1}{SF} \frac{AT_o}{2N_0} \right), & \text{for uplink (17)}
    \end{cases}
\end{align*}
\]

With the above equalization weights, MMSEC equalization is performed together with STTD decoding as in Eq. (13).

4. Simulation Results

Table 2 shows the computer simulation condition. We assume MC-CDMA using \( N_c=256 \) subcarriers with carrier spacing of \( 1/T_o \), GI of \( T_o=T/8 \) (i.e., \( T_o=32 \)), and ideal coherent BPSK data-modulation. IFFT and FFT sampling period \( AT \) is \( \Delta T=T/256 \). A frequency-selective Rayleigh fading channel
having $L=16$-path uniform power delay profile with $	au_l = 2\Delta T$ for downlink ( $	au_l^u = 2\Delta T, u = 0 \sim U - 1$, for uplink), and the normalized maximum Doppler frequency $f_D T = 0.01$ is assumed. Uncorrelated, time-varying Rayleigh faded paths are generated using Dent’s model [6].

<table>
<thead>
<tr>
<th>Data modulation</th>
<th>Coherent BPSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of subcarriers</td>
<td>$N_c = 256$</td>
</tr>
<tr>
<td>Effective symbol length</td>
<td>$T_s = 256\Delta T$</td>
</tr>
<tr>
<td>GI</td>
<td>$T_g = 32\Delta T$  $(T_g/T_s = 1/8)$</td>
</tr>
<tr>
<td>Spreading factor</td>
<td>$SF = 1 \sim 256$</td>
</tr>
</tbody>
</table>

Table 2: Simulation condition

4.1. Downlink

Figure 2 plots the downlink BER performance as a function of the average received signal energy per bit-to-the AWGN power spectrum density ratio $(E_b/N_0)$, $E_b/N_0 = (AT_s/N_0)(1 + T_g/T_s)$, for $SF = 32$ with the number $U$ of communicating users as a parameter. For reference, the BER with MMSEC equalization but for no diversity and that for 2-antenna receive diversity combined with MMSEC equalization as in [7] are also plotted. The BER performance worsens with the increase in the number of users due to increasing MUI. However, it is seen that the STTD achieves the performance of 2-antenna receive diversity but with a penalty of 3dB.

In the downlink, all users’ signal transmissions are synchronous and orthogonal spreading codes can be used to reduce the MUI. This suggests that as many as $SF$ users can be multiplexed on the downlink without significant performance degradation. Figure 3 plots the BER as a function of the average received $E_b/N_0$ with $SF$ as a parameter when the number $U$ of users is the same as $SF$. $SF = U = 1$ corresponds to the well-known orthogonal frequency division multiplexing (OFDM). It is interesting to note that the BER performance of MC-CDMA with $SF > 1$ with $U = SF$ is seen to be better than that of $SF = 1$ in spite of the increase in MUI. This is because as the value of $SF$ increases the increase in frequency diversity effect becomes large enough to offset the increase in MUI and also provide additional improvement [8]. With STTD, the average received $E_b/N_0$ for a BER of $10^{-4}$ is 8dB less for $SF = 256$ than that for $SF = 1$. The no diversity and 2-antenna receive diversity curves are also plotted. It is again seen that the STTD achieves the performance of 2-antenna receive diversity but with a 3dB penalty.

4.2. Uplink

In the uplink case, the orthogonality among the users cannot be maintained in a frequency selective channel. Hence, a BER error floor occurs when more than one user are in communication at the same time. The equalization weights were derived in Sect. 3 for the case when the users are synchronous. However, in actuality, the users’ transmitting timings are asynchronous. It was found from our preliminary simulation that the BER performances for the synchronous and asynchronous cases are almost identical. Fig. 4 plots the uplink BER performance as a function of the average received $E_b/N_0$ for $SF = 32$ with $U$ as a parameter for the asynchronous case when the users’ timing are uniformly distributed over $0 \sim T$. For reference, the BER performances for no diversity with MMSEC equalization and 2-antenna receive diversity combined with MMSEC equalization are also plotted. Using STTD with MMSEC equalization improves the performance, however the BER floor still exists.
5. Conclusion

In this paper, the STTD decoding combined with MMSEC equalization was presented for MC-CDMA and the equalization weights were derived where the weights are optimum in the sense of minimizing the MSE for each subcarrier. It was found that the equalization weights for downlink and uplink are different. From computer simulation, it was confirmed that the BER performance with STTD is always better than that with no diversity. STTD with MMSEC equalization using the weights derived in this paper provides the same diversity order as 2-antenna receive diversity with MMSEC equalization but with 3dB performance penalty. The uplink BER performance can also be improved with STTD, however the error floor still exists.

References