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Frequency-Domain Eigenbeam-SDM and Equalization for Single-Carrier Transmissions

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SUMMARY In mobile communications, the channel consists of many propagation paths resulting in a severely frequency-selective fading channel. The frequency-domain equalization (FDE) can take advantage of the channel selectivity and improve the bit error rate (BER) performance of the single-carrier (SC) transmission. Recently, multi-input multi-output (MIMO) multiplexing is gaining much attention for achieving very high speed data transmissions with the limited bandwidth. Eigenbeam space division multiplexing (E-SDM) is known as one of MIMO multiplexing techniques. In this paper, we propose frequency-domain SC E-SDM for SC transmission. In frequency-domain SC E-SDM, the orthogonal transmission channels to transmit different data in parallel are constructed at each orthogonal frequency. At a receiver, FDE is used to suppress the inter-symbol interference (ISI). In this paper, the transmit power allocation and adaptive modulation based on the equivalent channel gains after performing FDE are applied. The BER performance of the frequency-domain SC E-SDM in a severe frequency-selective Rayleigh fading channel is evaluated by computer simulation.

key words: MIMO multiplexing, frequency-domain SC E-SDM, MMSE-FDE, single-carrier transmission

1. Introduction

Broadband multimedia services with 100 Mbps–1 Gbps are demanded in the next generation mobile communication systems [1]. For such high speed data transmission, the mobile radio channel consists of many propagation paths with different time delays, resulting in a severely frequency-selective fading channel [2]. In such a channel, the performance of single-carrier (SC) transmission significantly degrades due to severe inter-symbol interference (ISI). Multi-carrier transmission technique, i.e., orthogonal frequency division multiplexing (OFDM) [3] and multi-carrier code division multiple access (MC-CDMA) [4], [5], has been considered as a promising transmission technique. However, recently, it has been shown in [6]–[8] that the frequency-domain equalization (FDE) based on minimum mean square error (MMSE) criterion can significantly improve the bit error rate (BER) performance of the SC transmission. On the other hand, 2-dimension temporal-spatial equalization in the time-domain [9] is well known. MMSE-FDE is a linear operation in the frequency-domain. However, 2-dimension temporal-spatial equalization in time-domain is a linear operation in the time-domain. In 2-dimension temporal-spatial equalization, it is necessary to perform the convolution operation in the time-domain. Therefore, there is an advantage that MMSE-FDE has lower complexity than 2-dimension temporal-spatial equalization. Therefore, in this paper, we consider a SC transmission with MMSE-FDE.

Although very high speed data transmissions are demanded in next generation systems, the available bandwidth is limited. Therefore, highly spectrum-efficient transmission techniques are required. Recently, multi-input multi-output (MIMO) multiplexing [10] has been attracting much attention. There are two types of MIMO multiplexing. One is the space division multiplexing (SDM) [11], [12], where different transmit antennas transmit different data streams simultaneously using the same carrier frequency. At a receiver, it is necessary to separate the signals transmitted from different antennas. The other is the eigenbeam-SDM (E-SDM) [13], [14], in which several orthogonal channels are constructed based on the MIMO channel information shared by the transmitter and the receiver. In E-SDM, there is no interference among different data streams since they are transmitted over orthogonal channels. Therefore, E-SDM can be expected to provide better transmission performance than SDM. By applying the transmit power allocation and adaptive modulation, further performance improvement can be expected.

SDM and E-SDM have been researched vigorously. However, in most of the researches done, OFDM transmission was considered. Recently, we proposed frequency-domain SC SDM [15]. However, we haven’t investigated about SC E-SDM yet. Therefore, in this paper, we propose frequency-domain SC E-SDM for SC transmission, which constructs orthogonal channels in frequency-domain to transmit different data streams simultaneously and applies FDE based on the MMSE criterion to achieve the frequency diversity effect. Since, in the SC transmission, each data symbol is spread over all subcarriers, the transmitted data symbol can be correctly recovered by MMSE-FDE if a large number of subcarriers do not fade simultaneously. This is called the frequency diversity effect. The BER performance of frequency-domain SC E-SDM is evaluated by computer simulation to compare SDM using MMSE-FDE and OFDM E-SDM.

The remainder of this paper is organized as follows. Section 2 describes the proposed frequency-domain SC E-SDM and equalization. The power allocation and adaptive modulation are presented in Sect. 3. Section 4 presents the computer simulation results for the BER performance. Sec-

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tion 5 concludes this paper.

2. Frequency-Domain SC E-SDM

Figure 1 shows the transmitter/receiver structure of \((N_t, N_r)\) frequency-domain SC E-SDM with MMSE-FDE, where \(N_t\) is the number of transmit antennas and \(N_r\) is the number of receiver antennas.

2.1 Transmitted Signal

At the transmitter, binary information sequence is converted into \(C(\leq \min(N_t, N_r))\) parallel sequences by serial-to-parallel (S/P) conversion. The \(c\)th binary information sequence is transformed into the data modulated symbol sequence and divided into a sequence of \(N_c\)-symbol signal blocks. The \(C\) signal blocks to be transmitted via \(C\) orthogonal channels are represented using the vector representation as

\[
\mathbf{d}(t) = \begin{bmatrix} d_0(t) & \cdots & d_{C-1}(t) \end{bmatrix}^T, \quad t = 0 \sim N_c - 1,
\]

which will be transmitted from the \(n\)th antenna. The time-domain \(N_t\)-by-1 transmit signal vector is represented by

\[
\mathbf{\bar{d}}(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \mathbf{\bar{D}}_n(k) \exp\left(j2\pi \frac{k t}{N_c}\right),
\]

where \(\mathbf{\bar{D}}_n(k)\) is the \(k\)th frequency component of the \(n\)th transmit signal, is obtained as

\[
\mathbf{\bar{D}}(k) = \mathbf{W}_t(k) \mathbf{D}(k).
\]

Then, \(N_c\)-point IFFT is applied to \(\{\mathbf{\bar{D}}_n(k); k = 0 \sim N_c - 1\}\) to obtain the time-domain signal \(\mathbf{d}_n(t)\) as

\[
\mathbf{d}_n(t) = \mathbf{\bar{d}}_n(t),
\]

which will be transmitted from the \(n\)th antenna. The time-domain \(N_t\)-by-1 transmit signal vector is represented by \(\mathbf{d}(t) = [d_0(t), \cdots, d_{N_t-1}(t)]^T, \quad t = 0 \sim N_c - 1\). As shown in Fig. 2, the last \(N_g\) symbols in each block are copied and inserted as a cyclic prefix into the guard interval (GI), which is placed at the beginning of each block. The GI is inserted to suppress the inter-block interference and make the received block as a circular convolution between the transmitted block and the channel impulse response. Therefore, the interference among orthogonal frequency components due to the inter-block interference is avoided. However, the spectrum distortion due to the ISI still remains in the received block after
the GI removal. The restoration of the transmitted signal spectrum (or equalization) is the task of MMSE-FDE.

Next, signal blocks are transmitted simultaneously from \( N_t \) transmit antennas using the same carrier frequency.

### 2.2 Received Signal

At the receiver, \( N_r \) transmitted signals are received by \( N_r \) antennas via a frequency-selective fading channel, which consists of \( L \)-propagation paths with different time delays. The received signal \( r_n(t) \) on the \( n \)th antenna at time \( t \) can be expressed as

\[
r_n(t) = \sum_{l=0}^{L-1} \sum_{n_t=0}^{N_t-1} h_{n,n_l,t} \tilde{a}_l(n_t - \tau_l) + n_n(t),
\]

where \( h_{n,n_l,t} \) and \( \tau_l \) represent the \( l \)th path gain between the \( n \)th receive antenna and \( n_t \)th transmit antenna and delay time, respectively. \( n_n(t) \) represents a zero-mean complex Gaussian noise process with variance \( 2\sigma^2 \) (= \( 2N_0/T \), where \( N_0 \) is the noise power spectrum density and \( T \) is the symbol duration).

After the removal of the GI, \( N_r \)-point FFT is applied to decompose the received signal block into \( N_r \) frequency components. The \( k \)th frequency component \( R_n(k) \) of the received signal on the \( n \)th antenna can be expressed as

\[
R_n(k) = \sum_{t=0}^{N_r-1} r_n(t) \exp\left(-j2\pi\frac{tk}{N_r}\right)
\]

\[
= \sum_{l=0}^{L-1} \sum_{n_t=0}^{N_t-1} H_{n,n_l,k} \tilde{a}_l(n_t) + \Pi_n(k),
\]

where \( H_{n,n_l,k} \) and \( \Pi_n(k) \) represent the complex channel gain and the noise component, respectively, at the \( k \)th frequency and they are given by

\[
\begin{cases}
H_{n,n_l,k} = \sum_{t=0}^{N_r-1} h_{n,n_l,t} \exp\left(-j2\pi\frac{tk}{N_r}\right) \\
\Pi_n(k) = \sum_{t=0}^{N_r-1} n_n(t) \exp\left(-j2\pi\frac{tk}{N_r}\right)
\end{cases}
\]

### 2.3 De-Multiplexing

The C-by-1 received signal vector \( \tilde{R}_k = [\tilde{R}_0(k), \cdots, \tilde{R}_{C-1}(k)]^T \) is obtained by multiplying the received signal \( R(k) = [R_0(k), \cdots, R_{C-1}(k)]^T \) by the C-by-\( N_r \) receive weight matrix \( W_r(k) \). Using the eigenvalue decomposition of the \( N_r \)-by-\( N_r \) channel matrix \( H(k) \), we obtain the transmit/receive weight matrices, \( W_r(k) \) and \( W_t(k) \), to construct the orthogonal channels. The eigenvalue decomposition of \( H(k) \) is expressed as

\[
H^H(k)H(k) = U(k)\Lambda(k)U^H(k),
\]

where \( U(k) \) is the \( N_r \)-by-C unitary matrix, \( \Lambda(k) = \text{diag}(\lambda_0(k), \lambda_1(k), \cdots, \lambda_{C-1}(k)) \) is the C-by-C diagonal matrix with \( \lambda_i(k) \) representing the \( i \)th eigenvalue of \( H(k) \), and \((\cdot)^T \) is the Hermitian transpose operation. From Eq. (7), \( W_r(k) \) and \( W_t(k) \) can be obtained as

\[
\begin{cases}
W_r(k) = U(k)P \\
W_t(k) = U^H(k)H^H(k)
\end{cases}
\]

where \( P = \text{diag}[\sqrt{2P_{c0}}, \cdots, \sqrt{2P_{cC-1}}] \) is the C-by-C transmit power matrix. \( P_c \) is the transmit power of the \( c \)th channel. \( P \) is determined by using the water filling theorem [16] based on the equivalent channel gain after equalization (which will be defined later).

For de-multiplexing the C transmitted signal blocks, \( R(k) \) is multiplied by the receive weight matrix \( W_r(k) \) to obtain \( \tilde{R}(k) \) as

\[
\tilde{R}(k) = W_r(k)R(k) = \Lambda(k)PD(k) + W_r(k)\Pi(k),
\]

Since \( \Lambda(k) \) and \( P \) are the diagonal matrix, the transmitted signal blocks can be de-multiplexed without suffering the interference from other antennas. From Eq. (9), we have

\[
\bar{R}_k(t) = [\sqrt{2P_c}\lambda_k D(k) + \sum_{n=0}^{N_r-1} W_{r,c,n}(k)\Pi_n(k),
\]

where \( \bar{R}_k(t) \) is the \( k \)th frequency component of the \( k \)th received signal, \( W_{r,c,n}(k) \) is the \( c \)th row and \( n \)th column component of \( W_r(k) \), the first term is the desired signal component and the second is noise component.

### 2.4 Frequency-Domain Equalization

Although de-multiplexing without suffering the interference from other antennas is done, the ISI still remains in the SC transmission. This ISI (within each block) produces the signal spectrum distortion resulting in the channel frequency-selectivity. We apply MMSE-FDE to suppress the ISI. After MMSE-FDE, we obtain

\[
\tilde{R}_k(t) = \sqrt{2P_c}\lambda_k D(k)
\]

\[
\begin{aligned}
&+ w_{\text{FDE},k} \sum_{n=0}^{N_r-1} W_{r,c,n}(k)\Pi_n(k),
\end{aligned}
\]

where \( \tilde{R}_k(t) \) is the \( k \)th frequency component of the \( k \)th received signal after MMSE-FDE and \( w_{\text{FDE},k} \) is the MMSE weight. We define the equalization error \( e_c(k) \) as

\[
e_c(k) = \tilde{R}_k(t) - \sqrt{2P_c}D(k).
\]

We want to find the equalization weight \( w_{\text{FDE},k} \) that minimizes the mean square error (MSE) \( \mathbf{E}[|e_c(k)|^2] \) for the given \( \lambda_k \). Following the steps shown in Ref. [7] and solving \( \partial\mathbf{E}[|e_c(k)|^2]/\partial w_{\text{FDE},k} = 0 \), the MMSE weight \( w_{\text{FDE},k} \) is obtained as

\[
w_{\text{FDE},k} = \frac{1}{\lambda_k + \left(P_c/(\sigma^2)\right)^{-1}}.
\]
In this paper, as we assume the ideal channel estimation, the equivalent channel gain after forming orthogonal channels is the eigenvalue, which is scalar, from Eq. (10). Therefore, MMSE-FDE weights are scalar too from Eq. (13).

After MMSE-FDE, \( N_c \)-point IFFT is applied to obtain the time-domain signal \( \tilde{r}_c(t) \) by using \( \{ \tilde{r}_c(k); k = 0 \sim N_c - 1 \} \) is given by

\[
\tilde{r}_c(t) = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \tilde{r}_c(k) \exp \left( j2\pi \frac{k}{N_c} \right)
\]

\[
= \sqrt{P_c} \sum_{k=0}^{N_c-1} w_{\text{FDE}, c}(k) \lambda_c(k) d_c(t)
\]

\[
+ \frac{1}{N_c} \sum_{k=0}^{N_c-1} w_{\text{FDE}, c}(k) \lambda_c(k) \left\{ \sum_{\tau=0}^{N_c-1} d_c(\tau) \exp \left( j2\pi (t-\tau) \frac{k}{N_c} \right) \right\}
\]

\[
+ \frac{1}{N_c} \sum_{k=0}^{N_c-1} w_{\text{FDE}, c}(k) \left( \sum_{n=0}^{N_c-1} W_{c,n}(k) \Pi_{n}(k) \right) \exp \left( j2\pi t \frac{k}{N_c} \right),
\]

where the first term is the desired signal component, the second the ISI component and the third the noise component. The equivalent channel gain of the \( c \)th signal is given by

\[
\hat{H}_c = \frac{1}{N_c} \sum_{k=0}^{N_c-1} w_{\text{FDE}, c}(k) \lambda_c(k).
\]

After parallel-to-serial (P/S) conversion, the received signal is data-demodulated to recover the transmitted binary information sequence.

3. Power Allocation and Adaptive Modulation

The transmit power and modulation level are determined block-by-block based on the equivalent channel gain. For the power allocation, the water filling theorem [16] is used to maximize the channel capacity. After the power allocation, the combination of the modulation level is decided. The adaptive modulation scheme to determine the combination of the modulation levels is applied based on the minimum BER criterion by using the Chernoff upper bound [2].

3.1 Power Allocation

The total channel capacity \( C_{\text{total}} \) of the \( C \) parallel orthogonal channels is given by [17]

\[
C_{\text{total}} = \sum_{c=0}^{C-1} \log \left( 1 + \gamma_c \right),
\]

where \( \gamma_c \) is the received signal power-to-Noise power ratio (SNR) of the \( c \)th channel and is given as [see Appendix A]

\[
\gamma_c = \frac{P_c}{\sigma_{\text{noise},c}^2} \frac{1}{N_c^2} \left\{ \sum_{k=0}^{N_c-1} w_{\text{FDE}, c}(k) \lambda_c(k) \right\}^2,
\]

where

\[
\sigma_{\text{noise},c}^2 = \frac{\sigma_{\text{noise},c}^2}{N_c} \sum_{k=0}^{N_c-1} w_{\text{FDE}, c}(k) \lambda_c(k),
\]

is the noise power.

Generally, when the power allocation based on maximum channel capacity is performed, the Lagrange multiplier method is used. Using the Lagrange multiplier method, the power set \( \{ P_0, \cdots, P_{C-1} \} \) that maximizes \( C_{\text{total}} \) is determined under the total power constraint \( P_{\text{total}} = C \sum_{c=0}^{C-1} P_c \). However, since MMSE-FDE is used, it is quite difficult to find theoretically the best power allocation using the Lagrange multiplier method. The MMSE-FDE weight approaches the ZF weight as the value of \( \lambda_c(k) \) increases, while it approaches the MRC weight as the value of \( \lambda_c(k) \) decreases. Therefore, in this paper, although MMSE-FDE is actually used, the power allocation is done assuming ZF weight (which is better adapted to the first channel) and also assuming MRC weight (which is better adapted to the fourth channel). \( P_c \) is found as [see Appendix B]

\[
P_c = \max \left\{ \begin{array}{cc}
P_{\text{total}} \frac{1}{C} + \sigma_{\text{noise},c}^2 \sum_{c=0}^{C-1} \frac{(N_c-1)}{1} \lambda_c(k) \end{array} \right\}
\]

for ZF,

\[
P_c = \max \left\{ \begin{array}{cc}
P_{\text{total}} \sum_{c=0}^{C-1} \frac{1}{C} \lambda_c(k) \end{array} \right\}
\]

for MRC.

3.2 Adaptive Modulation

The conditional signal-to-interference plus noise power ratio (SINR) \( \gamma'_c \) of the \( c \)th channel is given as

\[
\gamma'_c = \frac{P_c}{\sigma_{\text{ISI},c}^2 + \sigma_{\text{noise},c}^2} \cdot \frac{1}{N_c^2} \left\{ \sum_{k=0}^{N_c-1} w_{\text{FDE}, c}(k) \lambda_c(k) \right\}^2,
\]

where \( \sigma_{\text{ISI},c}^2 \) is the ISI power of the \( c \)th channel, and can be shown as [see Appendix A]
Based on the Gaussian approximation of the ISI, we treat the sum of ISI and noise as a new Gaussian noise. The conditional BER $P_{b,c}(\gamma')$ for the $c$th channel is given as [2]

$$P_{b,c}(\gamma') = \alpha_c \cdot \text{erfc} \left( \frac{\gamma'}{\beta_c} \right),$$

(23)

where $\text{erfc}(.)$ is the complementary error function and $\alpha_c$ and $\beta_c$ are shown in Table 1 [2]. In this paper, the Chernoff upper bound of the BER is used for determining the modulation level. The BER upper bound for the $c$th channel is given as [2]

$$P_{b,c}(\gamma') = \alpha_c \text{erf} \left( \sqrt{\frac{\gamma'}{\beta_c}} \right) \leq 2\alpha_c \exp \left( -\frac{\gamma'}{\beta_c} \right).$$

(24)

For the modulation using $m_c$ bits per symbol, the BER upper bound averaged over $C$ orthogonal channels is given as

$$P_b(\gamma'_c) = \frac{\sum_{c=0}^{C-1} m_c P_{b,c}(\gamma'_c)}{\sum_{c=0}^{C-1} m_c} \leq \frac{1}{\eta} \sum_{c=0}^{C-1} 2\alpha_c m_c \exp \left( -\frac{\gamma'_c}{\beta_c} \right),$$

(25)

where $\eta = \sum_{c=0}^{C-1} m_c$ is the spectrum efficiency in bps/Hz.

The modulation level is determined as follows. After the performing power allocation using the water filling theorem, using Eq. (19) or (20), the optimum combination of the modulation levels $(m_0, \cdots, m_{C-1})$ which minimizes the BER upper bound is found for the given spectrum efficiency $\eta$.

4. Computer Simulation

The simulation parameters are given in Table 2. We assume an information bit sequence of $K=1024$ bits. Turbo coding [18] is well known as a powerful channel coding and has been used in the present third generation mobile communication systems [19]. A rate-1/3 turbo encoder consisting of two (13, 15) recursive systematic convolutional (RSC) component encoders with a constraint length of 4 is assumed.

Table 1  $\alpha_c$ and $\beta_c$.

<table>
<thead>
<tr>
<th>Data modulation</th>
<th>$\alpha_c$</th>
<th>$\beta_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPSK</td>
<td>1/2</td>
<td>1</td>
</tr>
<tr>
<td>QPSK</td>
<td>1/2</td>
<td>2</td>
</tr>
<tr>
<td>8PSK</td>
<td>1/3</td>
<td>1/sin(π/8)</td>
</tr>
<tr>
<td>16QAM</td>
<td>3/8</td>
<td>10</td>
</tr>
<tr>
<td>64QAM</td>
<td>7/24</td>
<td>42</td>
</tr>
<tr>
<td>256QAM</td>
<td>15/64</td>
<td>170</td>
</tr>
</tbody>
</table>

Table 2  Simulation parameters.

<table>
<thead>
<tr>
<th>No. of information bits</th>
<th>1024bits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data modulation</td>
<td>BPSK/QPSK/8PSK/16QAM/64QAM/256QAM</td>
</tr>
<tr>
<td>No. of FFT/IFFT points</td>
<td>$N_c=256$</td>
</tr>
<tr>
<td>GI</td>
<td>$N_c=32$</td>
</tr>
<tr>
<td>Number of antennas</td>
<td>$(N_xN_y)=(2,2), (4,4)$</td>
</tr>
<tr>
<td>Fading channel</td>
<td>$L=16$-path Rayleigh fading</td>
</tr>
<tr>
<td></td>
<td>Exponential power delay profile with decay factor $\alpha=0$, 6dB</td>
</tr>
<tr>
<td>Channel estimation</td>
<td>Ideal</td>
</tr>
<tr>
<td>Feed back delay</td>
<td>None</td>
</tr>
</tbody>
</table>

Fig. 3  Effect of power allocation/adaptive modulation.

The resulting two parity sequences are punctured to obtain rate-1/2 turbo codes. Log-MAP decoding with 8 iterations is assumed. $N_x$-$N_y$ channels are assumed to be independent frequency-selective quasi-static $L=16$-path Rayleigh fading channels (i.e., $f_0T \to 0$, where $T$ is the symbol length), each path having a symbol-spaced exponentially decaying power delay profile with decay factor $\alpha$. Ideal channel estimation is assumed. We assume no feedback delay of the channel information from the receiver to transmitter. We have compared the uncoded BER performances achievable by frequency-domain SC E-SDM with power allocation assuming the ZF weight and assuming the MRC weight (it should be noted that MMSE-FDE is actually used). Since almost no performance difference was seen, we assume the ZF weight in the following simulation.

4.1 Effect of Power Allocation

Figure 3 plots the BER performances of $(4,4)$ frequency-domain SC E-SDM with and without power alloca-
tion/adaptive modulation as a function of the total transmitted energy-to-noise power spectrum density ratio SNR<sub>t</sub> (=P<sub>total</sub>/N<sub>0</sub>). The transmit power of each orthogonal channel is equal in the case of frequency-domain SC E-SDM without power allocation/adaptive modulation. The decay factor α is 0 dB and the spectrum efficiency η is 8 bps/Hz. It can be seen that the BER performance of the frequency-domain SC E-SDM with power allocation/adaptive modulation is much better than that without power allocation/adaptive modulation. Therefore, power allocation/adaptive modulation used in this paper is very effective.

4.2 Comparison with SDM with MMSE-FDE

First, we discuss the uncoded case. The uncoded BER performance of (N<sub>t</sub>, N<sub>r</sub>) frequency-domain SC E-SDM is plotted in Fig. 4 as a function of SNR<sub>t</sub>. For comparison, the BER performances of (N<sub>t</sub>, N<sub>r</sub>) SDM and (1, N<sub>r</sub>) SIMO, both with MMSE-FDE, are also plotted. In the frequency-domain SDM, MMSE-FDE is applied to separate the signals and suppress the ISI simultaneously [15].

It can be seen that frequency-domain SC E-SDM is superior to SDM and SIMO. When α=0 (6) dB, the required SNR<sub>t</sub> of (2, 2) frequency-domain SC E-SDM for the average BER=10<sup>-3</sup> is smaller by about 5 (7) dB than that of (2, 2) SDM. On the other hand, the required SNR<sub>t</sub> of (4, 4) frequency-domain SC E-SDM is smaller by about 6.5 and 9 dB than that of (4, 4) SDM when α=0 and 6 dB, respectively. This is because, in E-SDM, orthogonal channels are constructed, thereby producing no interference from other antennas and the adaptive power allocation/modulation is applied while, in SDM, MMSE-FDE cannot completely suppress the interference from other antennas and furthermore no adaptive power allocation/modulation is used. It can be seen that the BER performance with frequency-domain SC E-SDM is less sensitive to the channel frequency-selectivity (or α), since the ISI caused by the frequency-selectivity is better suppressed by performing FDE as well as adaptive power allocation/modulation.

Frequency-domain SC E-SDM is more complex than SDM, since the construction of orthogonal channels using eigenvalue decomposition is necessary and the transmit power allocation and adaptive modulation are applied. For frequency-domain SC E-SDM, the number of FFT and IFFT operations is two, FDE operation is one and the weight multiplication at the transmitter and receiver is one per transmitted data block. Therefore, the number of complex multiply operations is (2C + N<sub>t</sub> + N<sub>r</sub>)N<sub>c</sub>log<sub>2</sub>N<sub>c</sub> + CN<sub>t</sub>N<sub>r</sub>(CN<sub>t</sub> + N<sub>r</sub> + 1) for frequency-domain SC E-SDM. On the other hand, for frequency-domain SDM, the number of FFT and IFFT operation is one and FDE operation is one per transmitted data block. Therefore, the number of complex multiply operations is (N<sub>t</sub> + N<sub>r</sub>)N<sub>c</sub>log<sub>2</sub>N<sub>c</sub> + N<sub>t</sub>N<sub>r</sub>N<sub>c</sub><sup>2</sup> for frequency-domain SC SDM. In the case of N<sub>t</sub> = N<sub>r</sub> = C = 4 and N<sub>c</sub>=256, the number of complex multiply operations is 66,560 for frequency-domain SC E-SDM, and 32,768 for frequency-domain SC SDM. In this case, frequency-domain SC E-SDM has about two times larger number of complex multiply operations than frequency-domain SC SDM.

The turbo coded BER performance of (4, 4) frequency-domain SC E-SDM with spectrum efficiency of 8 bps/Hz is plotted in Fig. 5. Similar to the uncoded case, frequency-domain SC E-SDM is also superior to SDM and SIMO. In the case of R=1/2, the required SNR<sub>t</sub> of frequency-domain SC E-SDM, for the average BER=10<sup>-4</sup>, is smaller by about 2 (3.5) dB than that of SDM when α=0 (6) dB. Frequency-
domain SC E-SDM provides the better performance than SDM. In the turbo coded case, the BER performance of the frequency-domain SC E-SDM is sensitive to the channel frequency-selectivity, since interleaving effect is sensitive to the channel frequency-selectivity.

4.3 Comparison with OFDM E-SDM

First, we compare achievable BER performances of frequency-domain SC E-SDM and OFDM E-SDM. The same subcarrier-by-subcarrier power allocation/adaptive modulation as frequency-domain SC E-SDM is applied to OFDM E-SDM system. The BER performances of both E-SDM are plotted in Fig. 6 as a function of SNR. Frequency-domain SC E-SDM provides BER performance similar to OFDM E-SDM for the uncoded case. However, the former is slightly inferior to the latter for the turbo coded case; about 1 dB larger SNR is required for BER=10^-3. Turbo coding can improve the BER performances of both frequency-domain SC E-SDM and OFDM E-SDM. However, frequency-domain SC E-SDM is slightly inferior to OFDM E-SDM. In SC transmission, since \( \lambda_C(k) \) changes in the frequency-domain and hence, the ISI is produced. This can be reduced by MMSE-FDE, but cannot be completely removed. A slight performance degradation of frequency-domain SC E-SDM compared to OFDM E-SDM is due to the residual ISI after MMSE-FDE.

Next, we discuss the computational complexities of frequency-domain SC E-SDM and OFDM E-SDM in terms of the number of complex multiply operations and that of eigenvalue decomposition operations, respectively. As frequency-domain SC E-SDM and OFDM E-SDM form orthogonal channels at each frequency, the number of eigenvalue decomposition operations of \( N_f \)-by-\( N_f \) matrix size is \( N_c \) per transmitted data block of \( N_c \) symbols (in the case of OFDM, \( N_c \) is the number of subcarriers). On the other hand, the number of complex multiply operations is \((2C + N_f + N_c)N_f \log_2 (N_c + N_f^2 + 1)\) for frequency-domain SC E-SDM and \((N_f + N_c)N_f \log_2 (N_c + N_f^2 + 1)\) for OFDM E-SDM. Frequency-domain SC E-SDM requires \( C \) times larger number of FFT/IFFT operations per transmitted data block than OFDM E-SDM. Therefore, the computational complexity of frequency-domain SC E-SDM is almost the same as OFDM E-SDM.
For both E-SDM schemes, eigenvalue decomposition of $N_t$-by-$N_t$ matrix is required for each subcarrier at the receiver. The eigenvalue and $N_t$-by-C Unitary matrix must be feedback to the transmitter for each subcarrier to form orthogonal channels for each subcarrier. As a consequence, both E-SDM need the same amount of feedback information.

Finally, we discuss PAPR of both E-SDM. We found that the complementary cumulative distribution function (CCDF) of PAPR is almost insensitive to the channel decay factor $\alpha$. Figure 7 plots the CCDF curve for $\alpha = 0$ dB. It can be seen that frequency-domain SC E-SDM has significantly less PAPR than OFDM E-SDM although its BER performance is slightly inferior to OFDM E-SDM for the coded case see Fig. 6.

The above discussions show the superiority of our proposed frequency-domain SC E-SDM.

5. Conclusions

In this paper, we proposed frequency-domain SC E-SDM that constructs the orthogonal channels in the frequency-domain and performs MMSE-FDE to suppress the ISI. The power allocation based on the water filling theorem and the adaptive modulation using the Chernoff upper bound were applied. The average BER performance in a frequency-selective Rayleigh fading channel was evaluated by computer simulation. It was shown that frequency-domain SC E-SDM provides BER performance superior to SDM. Frequency-domain SC E-SDM was compared with OFDM E-SDM. It was shown that frequency-domain SC E-SDM achieves slightly worse BER performance than OFDM E-SDM for the coded case; however, it has significantly less PAPR than OFDM E-SDM.

Theoretical BER analysis for frequency-domain SC E-SDM with adaptive modulation is very difficult if not impossible. Therefore, in this paper, the BER performance was evaluated by computer simulation only. The theoretical BER analysis is left for future work. In this paper, we applied the power allocation based on the water filling theorem to maximize the channel capacity and the adaptive modulation to minimize the BER under the constraint of the frequency efficiency (bps/Hz). How proposed frequency-domain SC E-SDM improves the throughput performance other than the BER performance is left for a future work.

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References


Appendix A: Derivation of $\sigma^2_{\text{noise},c}$ and $\sigma^2_{\text{ISI},c}$

The noise component $\mu_{\text{noise},c}(t)$ of Eq. (14) can be represented as

$$\mu_{\text{noise},c}(t) = \frac{1}{N_c} \sum_{k=0}^{N_t-1} w_{\text{FDE},c}(k) \sum_{n=0}^{N_c-1} w_{c,n}(k) \Pi_{n}(k).$$
we have
\[
\mu_{\text{ISL}_c}(t) = \frac{\sqrt{2P_c}}{N_c} \sum_{k=0}^{N_c-1} u_{\text{FDE},c}(k)\lambda_c(k) \cdot \left( \sum_{\tau_0}^{N_c-1} d_c(\tau) \exp \left( j2\pi \frac{t-\tau}{N_c} \right) \right).
\]  

(A-2)

Using Eq. (A-1), the variance \(2\sigma^2_{\text{noise},c}\) of \(\mu_{\text{noise},c}(t)\) is given by
\[
2\sigma^2_{\text{noise},c} = E[|\mu_{\text{noise},c}(t)|^2] = \frac{1}{N_c} \sum_{k=0}^{N_c-1} \sum_{k'=0}^{N_c-1} u_{\text{FDE},c}(k)u_{\text{FDE},c}(k') \times \left( E \left[ \Pi_{n_c}(k)\Pi_{n_c}'(k') \right] \times \exp \left( j2\pi \frac{k-k'}{N_c} \right) \right).
\]  

(A-3)

Since
\[
E \left[ \Pi_{n_c}(k)\Pi_{n_c}'(k') \right] = 2N_c\sigma^2\delta(n_c-n_c'),
\]  

(A-4)

we have
\[
2\sigma^2_{\text{noise},c} = \frac{2\sigma^2}{N_c} \sum_{k=0}^{N_c-1} \sum_{k'=0}^{N_c-1} u_{\text{FDE},c}(k)u_{\text{FDE},c}(k') \times \left( \sum_{n_c=0}^{N_c-1} w_{n_c,n_c}(k)w_{n_c,n_c}'(k') \exp \left( j2\pi \frac{k-k'}{N_c} \right) \right) = \frac{2\sigma^2}{N_c} \sum_{k=0}^{N_c-1} u_{\text{FDE},c}(k)\lambda_c(k).
\]  

(A-5)

The variance \(2\sigma^2_{\text{ISL}_c}\) of \(\mu_{\text{ISL}_c}(t)\) is given by
\[
2\sigma^2_{\text{ISL}_c} = E[|\mu_{\text{ISL}_c}(t)|^2] = \frac{2P_c}{N_c^2} \sum_{k=0}^{N_c-1} \sum_{k'=0}^{N_c-1} u_{\text{FDE},c}(k)\lambda_c(k)u_{\text{FDE},c}(k')\lambda_c(k') \times \left( E[d_c(\tau)d_c^*(\tau')] \times \exp \left( j2\pi \frac{t-\tau}{N_c} \right) \exp \left( -j2\pi \frac{k-k'}{N_c} \right) \right).
\]  

(A-6)

Since we assume \(E[d_c(\tau)d_c^*(\tau')] = \delta(\tau-\tau')\), we have
\[
2\sigma^2_{\text{ISL}_c} = \frac{2P_c}{N_c^2} \sum_{k=0}^{N_c-1} \sum_{k'=0}^{N_c-1} u_{\text{FDE},c}(k)\lambda_c(k)u_{\text{FDE},c}(k')\lambda_c(k') \times \left( \sum_{\tau_0}^{N_c-1} \exp \left( j2\pi (k-k') \frac{t-\tau}{N_c} \right) \right)
\]  

\[
= \frac{2P_c}{N_c^2} \sum_{k=0}^{N_c-1} \sum_{k'=0}^{N_c-1} u_{\text{FDE},c}(k)\lambda_c(k)u_{\text{FDE},c}(k')\lambda_c(k') \times \left( (N_c\delta(k-k') - 1) \right)
\]  

\[
= \frac{2P_c}{N_c^2} \sum_{k=0}^{N_c-1} \left( u_{\text{FDE},c}(k)\lambda_c(k) \right)^2 - \frac{2P_c}{N_c^2} \left( \sum_{k=0}^{N_c-1} u_{\text{FDE},c}(k)\lambda_c(k) \right)^2.
\]  

(A-7)

Appendix B: Derivation of \(P_c\) 
ZF and MRC-FDE weights can represented as
\[
w_{\text{FDE},c} = \begin{cases} \frac{1}{\lambda_c(k)} & \text{for ZF} \\ \lambda_c(k) & \text{for MRC} \end{cases}.
\]  

(A-8)

Using Eq. (A-8), the \(\gamma_c\), which is SNR of the \(c\)th channel, can be represented as
\[
\gamma_c = \left( \frac{N_cP_c}{\sigma^2} \sum_{k=0}^{N_c-1} \lambda_c(k) \right)^2.
\]  

(A-9)

Using the Lagrange multiplier method under the total power constraint \(P_{\text{total}} = \sum_{c=0}^{C-1} P_c\), \(P_c\) can be taken. Using Eq. (16), (A-9) and \(P_{\text{total}} = \sum_{c=0}^{C-1} P_c\), evaluation function \(J\) can be represented as
\[
J = \left( \frac{1}{\sigma^2} \sum_{c=0}^{C-1} \lambda_c(k) \right)^2 - \frac{1}{\sigma^2} \sum_{c=0}^{C-1} P_c.
\]  

(A-10)

where \(\eta\) is undetermined coefficient. Equation (A-10) are differentiated by \(P_c\) as
Using Eq. (A-11) and $\frac{\partial J}{\partial P_c} = 0$, $P_c$ can be expressed as:

$$
P_c = \begin{cases}
\frac{P_{\text{total}}}{C} + C \sum_{c=0}^{C-1} \frac{1}{N_c} \sum_{k=0}^{N_c-1} \frac{1}{\lambda_c(k)} - \frac{\sigma^2}{N_c} \sum_{k=0}^{N_c-1} \frac{1}{\lambda_c(k)} & \text{for ZF} \\
\frac{P_{\text{total}}}{C} + N_c \sigma^2 \left( C \sum_{c=0}^{C-1} \frac{1}{N_c} \sum_{k=0}^{N_c-1} \frac{1}{\lambda_c(k)} \right) - N_c \sigma^2 \left( \sum_{k=0}^{N_c-1} \frac{1}{\lambda_c(k)} \right)^2 & \text{for MRC}
\end{cases}
$$

(A-12)

However, using Eq. (A-12), $P_c$ becomes negative occasionally. Therefore, $P_c$ is represented as:

$$
P_c = \begin{cases}
\max \left( \frac{P_{\text{total}}}{C} + \sigma^2 \frac{1}{N_c} \sum_{c=0}^{C-1} \sum_{k=0}^{N_c-1} \frac{1}{\lambda_c(k)} \right) & \text{for ZF} \\
\max \left( \frac{P_{\text{total}}}{C} + N_c \sigma^2 \frac{1}{C} \sum_{c=0}^{C-1} \sum_{k=0}^{N_c-1} \frac{1}{\lambda_c(k)} \right) & \text{for MRC}
\end{cases}
$$

(A-13)

since $P_c$ is always the value of 0 or more.