Downlink Capacity Comparison of MMSE-SVD and BD-SVD for Cooperative Distributed Antenna Transmission using Multi-user Scheduling

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Abstract—Cooperative distributed antenna transmission (CDAT) is a promising transmission technique to achieve a high link capacity by exploiting the spatial distribution of antennas over a macro-cell area. Recently, we proposed a minimum mean square error filtering combined with singular value decomposition (MMSE-SVD) for multi-user spatial multiplexing and showed that it can achieve higher sum capacity with lower computational complexity than BD-SVD which applies SVD after block diagonalization (BD). When using multi-user scheduling, the link capacity of MU-MIMO is affected by the combination of spatially multiplexed user equipments (UEs). In this paper, we evaluate by computer simulation the OFDM downlink capacities of MMSE-SVD and BD-SVD considering three types of multi-user scheduling (Max-SNR, Proportional Fair (PF) and Round Robin (RR)). We show that the MMSE-SVD can achieve higher link capacity with higher fairness than BD-SVD.

Keywords—distributed antenna; MU-MIMO; downlink; cooperative signal transmission; multi-user scheduling

I. INTRODUCTION

In the 5th generation (5G) networks, broader data services and higher link capacity than 4G networks are required [1]. However, due to the path loss, shadowing loss, and frequency-selective fading, it is difficult to achieve high quality services uniformly over the macro-cell area with limited bandwidth and transmit power [2]. As a promising solution, the authors have been studying the cooperative distributed antenna transmission (CDAT) [3, 4] to achieve a high link capacity by exploiting the spatial distribution of antennas over a macro-cell area. CDAT can mitigate the impact of path loss, shadowing loss, and fading since a certain number of antennas near a user equipment (UE) are adaptively selected according to the UE movement. High link capacity is achieved by introducing multi-user multi-input multi-output (MU-MIMO) into CDAT [5].

When applying MU-MIMO to the orthogonal frequency division multiplexing (OFDM) downlink transmission, it is necessary to mitigate the inter-antenna interference (IAI) and inter-user interference (IUI). Although the linear precoding technique can be used at macro base-station (MBS) side such as zero-forcing (ZF) and channel inversion [6], the received signal-to-noise power ratio (SNR) is deteriorated under a severe channel condition. The non-linear precoding technique such as vector perturbation (VP) [7] and Tomlinson-Harashima precoding (THP) [8] is attractive, but they require high computational complexity. Recently, block diagonalization combined with singular value decomposition (BD-SVD) was proposed [9] as a type of the linear filtering technique at both MBS side and UE side. In BD-SVD, the multi-user MIMO channel is transformed by BD into multiple single-user MIMO channels and then, SVD is applied for eigenmode transmission of spatially multiplexed data streams over each single-user MIMO channel. However, in BD-SVD, the diversity gain will reduce since the spatial degree of freedom is used for nulling IUI [10]. To avoid this problem by permitting IUI and IAI to remain to some extent, the authors proposed a minimum mean square error filtering combined with singular value decomposition (MMSE-SVD) for downlink OFDM [11].

In MU-MIMO, UEs are spatially multiplexed over the same bandwidth and interfere each other, hence the sum link capacity depends on the combination of spatially multiplexed UEs. Therefore, the sum link capacity is greatly affected by the multi-user scheduling. In this paper, we consider three types of scheduling (i.e., max-SNR, proportional Fair (PF) and round robin (RR)) [12] and evaluate by computer simulation the OFDM downlink capacity and fairness achievable by MMSE-SVD and BD-SVD in a multi-cell environment. The co-channel interference (CCI) from adjacent macro-cells as well as IAI and IUI affects the link capacity. MMSE-SVD is designed to take into account the received CCI at spatially multiplexed UEs when generating the MMSE filter.

The remainder of this paper is organized as follows. Section II introduces CDAT for downlink OFDM MU-MIMO, including MMSE-SVD and BD-SVD. Sect. III introduces three types of multi-user scheduling. Sect. IV presents computer simulation results. We will compare MMSE-SVD and BD-SVD in terms of link capacity, fairness, and computational complexity. Finally, Sect. V gives the concluding remarks.

Notations: [.]’ denotes the transpose operation, the Hermitian transpose operation, and the ensemble average operation, respectively. diag(·) and I_N denote the diagonal matrix and the N×N identity matrix, respectively. (x)° denotes max(0, x) operation.
II. CDAT FOR DOWNLINK OFDM MU-MIMO

A. Signal Representation

Fig. 1 shows the transmit/receive structure of CDAT for downlink OFDM MU-MIMO. Multi-user spatial multiplexing of $U$ UEs, each is equipped with $N_{uc}$ antennas, is considered. $N_{mbs}$ distributed antennas to simultaneously transmit the $N_{str}$ (≤ $N_{uc}$) data streams per UE using $N_{c}$ subcarriers (the data stream is indexed by $n_{str}$ (0–$N_{str}$-1)). Note that suppressions of IUI and IAI are necessary.

At MBS, the data symbol sequence to be transmitted to each UE is serial-to-parallel (S/P) converted to $N_{str}$ parallel streams each consisting of $N_{uc}$-symbol blocks (k=0–$N_{uc}$-1). The $u$th UE’s $N_{str}$×1 downlink transmit symbol vector is represented by $\mathbf{D}_{k}(u)\rightarrow [d_{u,0}(k),\cdots,d_{u,N_{uc}}(k),\cdots,d_{u,N_{uc}-1}(k)]^{T}$. $N_{mbs}$×1 transmit signal vector after transmit filtering can be expressed as

$$S(k) = \frac{2E}{T_{c}}W_{mbs}(k)\mathbf{D}(k) = \frac{2E}{T_{c}}W_{mbs}(k)\begin{bmatrix} \mathbf{D}_{k}(u) \end{bmatrix}$$

where $E_{c}$ is the transmit symbol energy and $T_{c}$ is the symbol duration. $W_{mbs}(k)=[W_{mbs,0}(k),\cdots,W_{mbs,N_{uc}}(k),\cdots,W_{mbs,N_{uc}-1}(k)]$ is the $N_{str}\times U\cdot N_{str}$ transmit filter matrix. The transmit filter matrices for MMSE-SVD and BD-SVD are respectively expressed as

$$W_{mbs}(k) = W_{mbs,0}(k),\cdots,W_{mbs,N_{uc}}(k),\cdots,W_{mbs,N_{uc}-1}(k)$$

$$= \begin{bmatrix} \mathbf{U}^{H}(k)\mathbf{H}(k) \end{bmatrix}$$

$$\times \left( \frac{E_{c}}{N_{o}} \right)^{-1} \frac{N_{uc}}{U\cdot N_{str}}\sum_{m=0}^{N_{str}} \left[ 1 + \lambda_{m}(u) \mathbf{I}_{N_{str}} \right]^{-1}$$

$$(k)$$

for MMSE (SVD)

$$W_{mbs}(k) = W_{bd-ord,0}(k),\cdots,W_{bd-ord,N_{uc}}(k),\cdots,W_{bd-ord,N_{uc}-1}(k)$$

$$(k)$$

for BD (SVD)

where $\mathbf{H}(k)=[\mathbf{H}_{0}^{T}(k),\cdots,\mathbf{H}_{u}^{T}(k),\cdots,\mathbf{H}_{U-1}^{T}(k)]^{T}$ is $U\cdot N_{uc}\times N_{mbs}$ downlink MU-MIMO channel matrix. $\mathbf{U}(k) = \text{diag}[\mathbf{u}_{0}(k),\cdots,\mathbf{u}_{U-1}(k)]$, and $\mathbf{u}_{u}(k)$ is obtained by applying SVD to the downlink channel between $N_{mbs}$ distributed antennas and $N_{uc}$ antennas of the $u$th UE as

$$\mathbf{u}_{u}(k) = \mathbf{U}_{u}(k)\lambda_{u}(k)^{1/2}$$

$$(k)$$

where $\mathbf{U}_{u}(k)$ and $\lambda_{u}(k)$ are unitary matrices consisting of left and right singular vectors of $\mathbf{H}_{u}(k)$ as its columns respectively. $\mathbf{A}_{u}(k)$ is the $N_{str}\times N_{str}$ diagonal matrix whose $n_{str}$th diagonal element $A_{u}(k;n_{str})$ contains the eigenvalue of the $n_{str}$th eigenmode. $\mathbf{U}^{H}(k)\mathbf{H}(k)$ can be viewed as the equivalent channel when each UE applies eigenmode reception (i.e. the $u$th UE uses $\mathbf{U}_{u}(k)$ as the receive filter matrix). Regarding BD-SVD, $\mathbf{V}_{u}(k)$ is BD weight and obtained by applying SVD to $[\mathbf{H}_{0}^{T}(k),\cdots,\mathbf{H}_{u}^{T}(k),\cdots,\mathbf{H}_{U-1}^{T}(k)]^{T}$ as follow.

$$\mathbf{V}_{u}(k) = \mathbf{U}_{u}(k)\lambda_{u}(k)^{1/2}$$

$$(k)$$

The equivalent channel matrix $\mathbf{H}_{u}(k)\mathbf{V}_{u}(k)$ after BD can be considered as a single-user MIMO channel matrix since IUI is completely removed. $\mathbf{V}_{u}(k)$ is obtained by applying SVD to $\mathbf{H}_{u}(k)\mathbf{V}_{u}(k)$ as follow.

$$\mathbf{H}_{u}(k)\mathbf{V}_{u}(k) = \mathbf{U}_{u}(k)\lambda_{u}(k)^{1/2}\mathbf{V}_{u}(k)$$

$$(k)$$

$$\mathbf{P}_{u,mbs}(k)=\text{diag}[\mathbf{P}_{mbs,0}(k),\cdots,\mathbf{P}_{mbs,N_{uc}-1}(k)],$$

and $\mathbf{P}_{mbs}(k)$ of size $U\cdot N_{str}\times U\cdot N_{str}$ represents the water filling based power allocation [13] across eigenmodes and subcarriers. The $n_{str}$th diagonal element of $\mathbf{P}_{mbs}(k)$ is given as

$$\mathbf{P}_{u}(k;n_{str}) = \begin{bmatrix} 1 \cdots 1 \end{bmatrix}$$

$$(k)$$

where $\lambda_{u}$ is a constant which is selected to preserve transmit power constraint. The $n_{str}$th diagonal element of $\mathbf{P}_{u,mbs}(k)$ is given by (6) by replacing $A_{u}(k;n_{str})$ with $A_{u}(k;n_{str})$ which is the eigenvalue of the $n_{str}$th eigenmode of $\mathbf{A}_{u}(k)^{1/2}$.
At the \(u\)th UE, CP removal is employed and then, each block is transformed into the frequency-domain receive signal block by \(N_{c}\)-point DFT. The \(N_{c}\times1\) frequency-domain received signal vector \(\mathbf{R}_u(k)\) at \(k\)th subcarrier is expressed as

\[
\mathbf{R}_u(k) = \mathbf{H}_u(k)\mathbf{\hat{s}}_u(k) + \mathbf{N}_u(k),
\]

where \(\mathbf{N}_u(k)\) is the \(N_{c}\times1\) noise vector whose elements are zero-mean complex-valued random variables having variance \(2N_{0}T_{s}\) with \(N_{0}\) being the one-sided power spectrum density of additive white Gaussian noise (AWGN). The \(N_{c}\times1\) frequency-domain soft-output symbol vector \(\mathbf{\hat{d}}_u(k)\) is obtained by applying the receive filter on \(\mathbf{R}_u(k)\) as

\[
\mathbf{\hat{d}}_u(k) = [\hat{d}_{u,0}(k), \ldots, \hat{d}_{u,N_{c}-1}(k)]^T,
\]

where \(\mathbf{W}_u(k)\) is the \(N_{c}\times N_{c}\) receive filter matrix. The receive filter matrices for MMSE-SVD and BD-SVD are respectively expressed as

\[
\mathbf{W}_{u,\text{MMSE}}(k) = \mathbf{U}_u^H(k) \quad \text{(for MMSE – SVD)} \quad \text{and} \quad \mathbf{W}_{u,\text{BD}}(k) = \mathbf{U}_u^H(k) \quad \text{(for BD – SVD)}.
\]

### B. Link Capacity

OFDM downlink capacity \(C_u\) (bps/Hz) of the \(u\)th UE is computed using Shannon capacity formula [2] as

\[
C_u = \sum_{k=0}^{N_c-1} \sum_{n=0}^{N_{cmt}-1} \log_2 \left(1 + \gamma_u(k; n_{am})\right),
\]

where \(\gamma_u(k; n_{am})\) denotes the instantaneous received signal-to-interference plus noise power ratio (SINR) at the \(k\)th subcarrier after eigenmode reception of the \(n_{am}\)th eigenmode of the \(u\)th UE and is expressed as

\[
\gamma_u(k; n_{am}) = \frac{E_s}{N_{0}} \left[ \mu_{I,am}^u(k; n_{am}) + \mu_{N,am}^u(k; n_{am}) + \mu_{N,am-\text{CCI}}^u(k; n_{am}) \right],
\]

where \(\mathbf{H}_u(k; n_{am}, u \cdot N_{am} + n_{am})\) represents the equivalent downlink channel gain considering the transmit filter and the receive filter for the \(u\)th UE’s, \(n_{am}\)th stream (i.e., the \((n_{am}, u \cdot N_{am} + n_{am})\)th element of \(\mathbf{H}_u(k) = \mathbf{W}_{u,\text{MMSE}}(k)\mathbf{H}_u(k)\mathbf{W}_{u,\text{BD}}(k)\)). \(\mu_{I,am}^u(k; n_{am})\), \(\mu_{N,am}^u(k; n_{am})\) and \(\mu_{N,am-\text{CCI}}^u(k; n_{am})\) are respectively the residual IAI, residual IUI, and noise plus co-channel interference (CCI) corresponding to the \(n_{am}\)th stream of the \(u\)th UE, given by

\[
\begin{align*}
\mu_{I,am}^u(k; n_{am}) &= \frac{E_s}{N_{0}} \sum_{n_{am}'} \sum_{n_{am}''} \left[ \mathbf{H}_u(k; n_{am}, u \cdot N_{am} + n_{am}') \right]^H, \\
\mu_{N,am}^u(k; n_{am}) &= \frac{E_s}{N_{0}} \sum_{n_{am}'} \sum_{n_{am}''} \left[ \mathbf{H}_u(k; n_{am}, u \cdot N_{am} + n_{am}') \right]^H, \\
\mu_{N,am-\text{CCI}}^u(k; n_{am}) &= \left[ 1 + \frac{I_{d}(0)}{N_{0}} \right] \sum_{n_{am}'} \sum_{n_{am}''} \mathbf{W}_{u,\text{BD}}(k; n_{am}, n_{am}'),
\end{align*}
\]

where \(\mathbf{W}_{u,\text{BD}}(k; n_{am}, n_{am}')\) is the \((n_{am}, n_{am}')\)th element of \(\mathbf{W}_{u,\text{BD}}(k)\).

### III. MULTI-USER SCHEDULING

In this paper, Max-SNR, PF, and RR are considered for multi-user scheduling. The multi-user scheduler choses \(U\) UEs from \(U_{\text{act}}\) active UEs in the macro-cell to communicate at the time-slot \(t\). The combination \(Y(t)\) of spatially multiplexed UEs at time-slot \(t\) is a proper subset of the set \(Y\) of active UEs \(\{(t)\subset Y, |Y|=U_{\text{act}}, |Y(t)|=U\}\).

In Max-SNR, the multi-user scheduler choses the combination of spatially multiplexed UEs which maximize the sum link capacity at the time-slot as

\[
Y(t) = \arg \max_{\{t|\subset Y\}} \sum_{u \in Y(t)} \tilde{C}_u(t, Y(t)),
\]

where \(\tilde{C}_u(t, Y(t))\) is the expected link capacity (bps/Hz) of \(u\)th UE of the combination \(Y(t)\). In this paper, it is assumed that the multi-user scheduler does not have the knowledge of the UEs selections at the adjacent macro-cells, hence \(\tilde{C}_u(t, Y(t))\) is calculated without considering the CCI (i.e., set \(I_{d}(u) = 0\) in Eq. (10-12)).

In PF, the multi-user scheduler choses the combination of spatially multiplexed UEs as

\[
Y(t) = \arg \max_{\{t|\subset Y\}} \sum_{u \in Y(t)} \frac{\tilde{C}_u(t, Y(t))}{\tilde{C}_u(t-1)},
\]

where \(\tilde{C}_u(t)\) is the average link capacity of \(u\)th UE at time-slot \(t\), and is expressed as

\[
\tilde{C}_u(t) = \begin{cases} 
\frac{1}{T_p} \sum_{t=0}^{T_p-1} \tilde{C}_u(t) + \frac{1}{T_p} \tilde{C}_u(t-1), & u \in Y(t) \\
\frac{1}{T_p} \sum_{t=0}^{T_p-1} \tilde{C}_u(t-1), & u \notin Y(t)
\end{cases},
\]

where \(T_p\) is the average period.

In RR, the multi-user scheduler choses the combination of spatially multiplexed UEs in order to assign the communication opportunity in turn.

In this paper, fairness index (FI) is defined as [14]

\[
\text{FI} = \frac{\left(\sum_{u=0}^{U_{\text{ave}}-1} \frac{1}{T_{\text{ave}}} \sum_{t=0}^{T_{\text{ave}}-1} C_u(t)\right)^2}{U_{\text{ave}} \sum_{u=0}^{U_{\text{ave}}-1} \frac{1}{T_{\text{ave}}} \sum_{t=0}^{T_{\text{ave}}-1} C_u(t)},
\]

where \(T_{\text{ave}}\) is the average period and \(T_{\text{ave}} \gg T_{PF}\).

### IV. MONTE-CARLO COMPUTER SIMULATION

#### A. Simulation Setting

A simple hexagonal cellular model with \(N_{\text{macro}} = 19\) distributed antennas located uniformly in each macro-cell with radius of \(R\) is assumed, as shown in Fig. 2. \(U_{\text{act}} = 19\) UEs equipped with \(N_{\text{UE}} = 2\) antennas are randomly located within a macro-cell. The cell of interest is isolated in single-cell condition, and is surrounded by 6 adjacent macro-cells in multi-cell condition. In each macro-cell, the multi-user scheduler choses \(U\) UEs according to each multi-user scheduling scheme. The number of data streams per UE is assumed to be \(N_{\text{am}} = 2\). \(N_{\text{sub}} \cdot U / N_{\text{UE}}\) distributed antennas are
selected from $N_{\text{macro}}$ distributed antennas in a descending order of the instantaneous received signal power level.

The broadband wireless channel is characterized by distance-dependent path loss, log-normally distributed shadowing loss, and multipath fading. Assuming that the channel is composed of $L=16$ distinct paths, the transfer function $H_u(k; n_u, n_{\text{th}})$ between the $n_{\text{th}}$ antenna of the $u$th UE in macro-cell and the distributed $n_{\text{th}}$ antenna can be represented as

$$H_u(k; n_u, n_{\text{th}}) = \sqrt{d_{n_u}} \exp \left( j \theta_{n_u} \right) \exp \left( -j \frac{2 \pi k R_{u u, n_{\text{th}}}}{N_u} \right) (17)$$

In this paper, the channel is assumed to be a Nakagami-Rice fading channel (i.e., dominant path to scattered path power ratio $K=10$ (dB)) when the distance $d_{n_u}$ between the $u$th UE in macro-cell and the $n_{\text{th}}$ distributed antenna is equal to or smaller than the small cell radius $R'=R'/\sqrt{9}$, and a Rayleigh fading channel (i.e., $K=0$) when $d_{n_u}$ is larger than $R'$. Path loss exponent $\alpha=3.5$, shadowing loss standard deviation $\eta_{n_u, n_{\text{th}}}=7$ (dB) are assumed. $\theta_{n_u}$ is the phase of dominant path and is assumed to be distributed uniformly. $\xi_{n_u, n_{\text{th}}}(l)$ and $\tau_{n_{\text{th}}}$ are respectively the complex-valued path gain and the time delay of the $l$th path with $E\left[ \sum_{l=1}^{L-1} \xi_{n_u, n_{\text{th}}}(l)^2 \right] = 1$ for all $u$, $n_u$, $n_{\text{th}}$. We assume a sample-spaced time delay (i.e., $\tau_{n_u, n_{\text{th}}}=l$ for all $u$, $n_u$, $n_{\text{th}}$). $H_u(k; n_{\text{th}})$ is $(u N_u)^{-1} n_{\text{th}}$th element of MU-MIMO channel matrix $H(k)$. $N_u=128$ points of DFT/IDFT, normalized transmit $E_u/N_0=0$(dB) and ideal channel estimation are assumed.

The CCI power spectrum densities, $I_j(u)$ is computed by

$$I_j(u) = \frac{E_u}{N_0} \sum_{k=1}^{K} \sum_{n \neq n_{\text{th}} = 0}^{N_u-1} \sum_{l=0}^{L-1} \sum_{c=0}^{C-1} E \left[ H_u(k; n_{\text{th}}, n_{\text{th}})(c) H_u(k; n_{\text{th}}, n_{\text{th}})(c) \right]$$

where $W_{n_{\text{th}}}(k; n_{\text{th}}, n_{\text{th}})(c)$ is $(n_{\text{th}}, u : N_{\text{thorm}}+n_{\text{th}})^{1}$th element of $\mathbf{w}_{n_{\text{th}}}(k)$ of $c$th adjacent macro-cell.

**B. Simulation Result**

Fig. 3 plots the average downlink sum capacity as a function of the average FI in single-cell condition. It is seen from Fig. 3 that the average downlink sum capacity increases as the number $U$ of spatially multiplexed UEs gets larger since the number of transmit streams increases. Max-SNR can achieve high downlink sum capacity, but the average FI reduces since particular UEs with a good channel condition are often assigned communication opportunities. On the other hand, RR and PF, which provide almost fair communication opportunities to all UEs, achieve higher average FI than Max-SNR. In RR and PF, MMSE-SVD can achieve higher downlink sum capacity with higher average FI than BD-SVD as $U$ gets larger. This is because that the equivalent channel gain after BD reduces in BD-SVD, while MMSE-SVD can avoid this problem by permitting IUI and IAI to remain to some extent. When $U=4$, in RR (PF), MMSE-SVD can achieve 6% (5%) higher downlink sum capacity and 14% (17%) higher average FI than BD-SVD.

Fig. 4 plots the average downlink sum capacity as a function of average FI in multi-cell condition. It is seen from Fig. 4 that the average downlink sum capacity reduces compared with single-cell condition because of CCI. In multi-cell condition, RR achieves higher downlink sum capacity than PF. It is thought that the expected link capacity in PF is calculated without considering the CCI so UEs near cell edge are often assigned communication opportunities. In RR and PF, MMSE-SVD can achieve higher downlink sum capacity with higher average FI than BD-SVD as $U$ gets larger as well as single-cell condition. When $U=4$, in RR (PF), MMSE-SVD can achieve 17% (5%) higher downlink sum capacity and 14% (12%) higher average FI than BD-SVD. In multi-cell condition, the link capacity superiority of MMSE-SVD to BD-SVD becomes larger since MMSE-SVD transmit filter is designed considering the CCI.

Next, we discuss the computational complexity. Fig. 5 plots the average number of complex multiplications for generating transmit filter per subcarrier filter as a function of $U$. It is seen from Fig. 5 that MMSE-SVD can reduce computational complexity compared to BD-SVD. When $U=2$ (4), the number of complex multiplications of MMSE-SVD is 49% (6.7%) of that of BD-SVD.

**V. Conclusion**

In this paper, we evaluate by computer simulation the OFDM downlink capacity and fairness when using MMSE-SVD and BD-SVD in a multi-cell environment. In MMSE-SVD, the received CCI at spatially multiplexed UEs are taken into account when generating the MMSE filter. Three types of scheduling, Max-SNR, PF and RR, were considered. It was confirmed that our proposed MMSE-SVD achieves higher sum link capacity with higher fairness than BD-SVD as the number
$U$ of spatially multiplexed UEs increases. It was shown that the stronger the CCI from adjacent macro-cells becomes, more pronounced the superiority of MMSE-SVD becomes while MMSE-SVD requires lower computational complexity.

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