Frequency-Domain Equalization for Single-Carrier SFBC Diversity in a Frequency-Selective Fading

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Abstract—In a high mobility environment, single-carrier (SC) space-frequency block coding (STBC) diversity achieves better bit error rate (BER) performance than SC space-time block coding (STBC) diversity. However, in a strong frequency-selective fading channel, the BER performance degrades due to the orthogonality distortion of SFBC codeword. In this paper, we propose a frequency-domain equalization (called robust FDE) suitable for SC-SFBC diversity in a frequency-selective fading. The robust FDE weights are jointly optimized based on minimum mean square error (MMSE) criterion taking into account channel variation within a SFBC codeword caused by frequency-selective fading. It is shown by computer simulation that proposed robust FDE always achieves BER performance superior to conventional FDE whose weights are determined without considering the channel frequency variation within a SFBC codeword.

Keywords-component; Space-frequency block coding, frequency-domain equalization, single-carrier transmission

I. INTRODUCTION

In the next generation wireless communications, broadband data services are demanded even in a high mobility environment. In such an environment, the transmit signal is subjected to a time and frequency doubly selective fading and bit error rate (BER) performance significantly degrades [1]. To overcome frequency-selective fading, single-carrier transmission with frequency-domain equalization (SC-FDE) [2-4] is attractive. SC transmission has lower peak-to-average power ratio (PAPR) property than orthogonal frequency division multiplexing (OFDM) transmission. The use of minimum mean square error (MMSE) based frequency-domain equalization (FDE) takes advantage of channel frequency-selectivity and obtains large frequency diversity gain [2-4].

The use of space-time block coding (STBC) diversity [5-7] in addition to FDE further improves the BER performance. However, in a high mobility environment, the orthogonality of STBC codeword is lost and hence, the BER performance degrades [8]. On the other hand, space-frequency block coding (SFBC) diversity [9-12] can avoid a problem resulting from strong time-selectivity of the channel. However, SFBC diversity can suffer from the frequency-selective fading.

In this paper, we propose a new FDE (called robust FDE) suitable for SC-SFBC diversity in a time and frequency doubly selective channel. The robust FDE weights are jointly optimized based on MMSE criterion taking into account channel variation within a STBC codeword caused by frequency-selective fading while the weights of the conventional FDE [11,12] are determined without considering the channel variation. We evaluate, by computer simulation, the BER performance when using the proposed robust FDE to show that the proposed robust FDE achieves a good BER performance in a time and frequency doubly selective fading channel.

The remainder of this paper is organized as follows. The SC-SFBC diversity is introduced in Sect. II. Sect. III describes the design of the robust FDE. Sect. IV discusses the computer simulation results, and Sect. V offers some conclusions.

II. SC-SFBC DIVERSITY

In this paper, SC-SFBC diversity using $J \times Q$ orthogonal space-time block codes (OSTBC) [7] is considered. It is assumed that the transmitter equips with $N_t$ antennas and the receiver has $N_r$ antennas, respectively. In SC-SFBC diversity using OSTBC, the number $J$ of transmit signal blocks and the number $Q$ of coded signal blocks and the code rate $J/Q$ depend on the number $N_t$ of transmit antennas summarized in Table I [7]. The code rate $J/Q$ of OSTBC decreases from 1.0 when $N_t>2$. On the other hand, an arbitrary number of receive antennas can be available without reducing the code rate $J/Q$.

<table>
<thead>
<tr>
<th>$N_t$</th>
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Fig. 1. Transmitter/receiver structures.
signal by SC-SFBC signal is transformed into the frequency-domain. At the receiver, after CP removal, the received signal is transmitted over a time and frequency doubly selective fading channel. The equalized received signal vector \( \hat{R} \) is given as

\[
\hat{R}(k) = \frac{2P}{N_t} H(k) S(k) + N(k),
\]

where \( S(k) = [S(0,k), \ldots, S(N_t-1,k)]^T \) is the coded frequency-domain transmit signal vector. \( H(k) = [H(0,k), \ldots, H(N_t-1,k)] \) with \( H(n,k) = \{H(n,0,k), \ldots, H(n,N_t-1,k)\} \) is the \( N_c \times N_t \) channel matrix and \( H(n,n,k) \) is the channel transfer function between the \( n \)-th transmit antenna and the \( n \)-th receive antenna. \( P_t \) denotes the transmit power. \( N(k) = [N(0,k), \ldots, N(N_t-1,k)]^T \) is the noise vector and \( N(n,k) \) is the zero mean complex valued additive white Gaussian noise (AWGN) having variance \( 2N_0T_s \) with \( N_0 \) and \( T_s \) being the single-sided power spectrum density of AWGN and the symbol duration, respectively. Then, the receive FDE is performed. The equalized received signal vector \( \hat{R}(k) = [\hat{R}(0,k), \ldots, \hat{R}(N_t-1,k)] \) is given as

\[
\hat{R}(k) = W(k) R(k),
\]

with \( W(k) = [W(0,k), \ldots, W(N_t-1,k)]^T \) with \( W(n,k) = [W(0,n,k), \ldots, W(N_t-1,n,k)] \) is the \( N_t \times N_c \) receive FDE weight matrix. The received signal after the receive FDE having \( N_t \) subcarrier components is decomposed into \( Q=2 \) received signal blocks having \( M \) subcarrier components each by subcarrier de-mapping. Then, OSFBC decoding is
carried out. The \( j \)th decoded received signal \( \{ \hat{D}_j(m) : m=0, \ldots, M-1 \} \), \( j=0, \ldots, J-1 \), is given as

\[
\hat{D}_j(m) = \hat{Y}_j(0, m) + \hat{Y}_j(1, m) \quad \text{... for } N_f=2
\]

where \( \hat{Y}_j(n, m) \) is the \( q \)th received signal after subcarrier demapping and given as \( \hat{Y}_j(n, m) = \hat{R}_j(n, e_j(m)) \). The decoded frequency-domain signal is transformed back to the time-domain signal by \( M \)-point IDFT and finally, data demodulation is carried out.

III. ROBUST FDE FOR A STRONG FREQUENCY-SELECTIVE FADING CHANNEL

In this paper, FDE weight is designed so as to minimize mean square error (MSE) between the transmit signal before OSTBC encoding and the received signal before OSTBC decoding. The MSE, \( e \), is given as

\[
e = \sum_{m=0}^{M-1} \left[ D_j(m) - \hat{D}_j(m) \right]^2 P^{-1} N_e, \quad (J/Q)^2
\]

(5)

From (1), (3) and (4), (5) can be rewritten as

\[
e = \sum_{n=0}^{N-1} \left[ H(n, e_j(m)) W(0, e_j(m)) + H^{\ast}(1, e_j(m)) W^{\ast}(1, e_j(m)) - 1 \right]^2
\]

\[
+ \left[ H(1, e_j(m)) W(1, e_j(m)) + H^{\ast}(0, e_j(m)) W^{\ast}(0, e_j(m)) \right] - 1 \]

\[
+ \sum_{n=0}^{N-1} \left[ H(n, e_j(m)) W(0, e_j(m)) - H^{\ast}(1, e_j(m)) W^{\ast}(1, e_j(m)) \right] \]

\[
+ \sum_{n=0}^{N-1} \left[ H(1, e_j(m)) W(1, e_j(m)) - H^{\ast}(0, e_j(m)) W^{\ast}(0, e_j(m)) \right] \]

\[
+ N \left( \frac{P}{Q} \right)^2 \sum_{n=0}^{N-1} \sum_{n=0}^{N-1} W(n, e_j(m)) \]

(6)

where \( N=N_0/T \) is the noise power.

A. Conventional FDE

In conventional FDE, the FDE weight is optimized assuming that the channel does not vary within a SFBC codeword (i.e., \( H(n, e_j(m)) \approx \cdots \approx H(n, e_{j-1}(m)) \)) [11,12]. In this case, the optimization problem is given as

\[
\begin{align*}
\left\{ W(0, e_j(m)), W(1, e_j(m)) \right\} &= \arg \min e \\
\text{s.t. } H(n, e_j(m)) &= \cdots = H(n, e_{j-1}(m)) = \bar{H}(n, m) 
\end{align*}
\]

(7)

where \( \bar{H}(n, m) = \sum_{j=0}^{J-1} H(n, e_j(m)) / Q \). By solving \( \partial e / \partial W(0, e_j(m)) = 0, \ldots, \partial e / \partial W(N_1 - 1, e_{j-1}(m)) = 0 \), the conventional FDE weight is obtained as

\[
W(n, e_j(m)) = W(n, e_j(m)) = \bar{H}^{\ast}(n, m) / \sum_{n=0}^{N-1} \left\| \bar{H}(n, m) \right\|^2 + N \left( \frac{J}{Q} \right)^2 \frac{P}{N}
\]

(8)

In the conventional FDE, the channel variation in the frequency-domain is not considered. Therefore, in a frequency-selective fading channel, BER performance degrades due to the interference caused by the channel variation.

B. Robust FDE

In this paper, we propose the robust FDE suitable for SC-SFBC diversity in a frequency-selective fading channel. In the proposed robust FDE, multiple FDE weights for a SFBC codeword are jointly optimized taking into account the channel frequency variation within a SFBC codeword (i.e., \( H(e_j(m)), H(0, e_j(m)) \cdots \approx H(e_{j-1}(m)) \)). The optimization problem for the robust FDE is given as

\[
\begin{align*}
\left\{ W(0, e_j(m)), W(1, e_j(m)) \right\} &= \arg \min e \\
\text{s.t. } W(0, e_j(m)), W(1, e_j(m)) \end{align*}
\]

(9)

By solving \( \partial e / \partial W(0, e_j(m)) = 0, \ldots, \partial e / \partial W(N_1 - 1, e_{j-1}(m)) = 0 \), the robust FDE weights are obtained as

\[
\begin{align*}
W(0, e_j(m)) &= H^{\ast}(0, e_j(m)) - H^{\ast}(1, e_j(m)) H_j(m) / \bar{H}_j(m) \\
W(1, e_j(m)) &= H^{\ast}(1, e_j(m)) - H^{\ast}(0, e_j(m)) H_j(m) / \bar{H}_j(m) \\
W(0, e_j(m)) &= H^{\ast}(0, e_j(m)) - H^{\ast}(1, e_j(m)) H_j(m) / \bar{H}_j(m) \\
W(1, e_j(m)) &= H^{\ast}(1, e_j(m)) - H^{\ast}(0, e_j(m)) H_j(m) / \bar{H}_j(m)
\end{align*}
\]

(10)

where

\[
\begin{align*}
\bar{H}_j(m) &= \sum_{n=0}^{N-1} H(n, e_j(m)) / Q \\
\bar{H}_j(m) &= \sum_{n=0}^{N-1} H(0, e_j(m)) + N \left( \frac{J}{Q} \right)^2 \frac{P}{N}
\end{align*}
\]

The second terms in denominator and numerator in (10) contribute to suppress the interference caused by the channel variation in the frequency-domain. In the case of weak frequency-selectivity of the channel (i.e., \( H(e_j(m)) \approx \cdots \approx H(e_{j-1}(m)) \)), (10) corresponds to conventional FDE weights given as (8).

IV. COMPUTER SIMULATION

We evaluate, by the computer simulation, the BER performance when using the proposed robust FDE. We consider QPSK data modulation. FFT block size \( N_f \) and CP length \( N_g \) are set to \( N_f=128 \) symbols and \( N_g=32 \) samples, respectively. The number \( N_f \) of transmit antennas and the number \( N_g \) of receive antennas are set to \( N_f=3 \) and \( N_g=2 \), respectively, as an example. The channel is assumed to be a time and frequency-selective fading. In this paper, we assume perfect CSI can be obtained at the receiver.
A. BER performance and PAPR performance

Fig. 4 shows the BER performance when using the proposed robust FDE as a function of transmit $E_b/N_0$. The power delay profile of channel is assumed to be symbol spaced $L=16$ path uniform power delay profile (the normalized rms delay spread $\tau_{rms}/T_s=4.61$). The normalized maximum Doppler frequency $f_d T_B$ is assumed to be $f_d T_B=1.5 \times 10^{-3}$, where $T_B=(N_c+N_g)T_s$ is FFT block duration. For the comparison, the performance when using the conventional FDE is also plotted in Fig. 4. It is seen from Fig. 4 that the proposed robust FDE can significantly improve the BER performance compared to the conventional FDE in a frequency-selective fading channel. This is because the proposed robust FDE weights are jointly optimized by taking into the channel variation in the frequency-domain and hence, it can suppress the interference caused by the orthogonality distortion of SFBC codeword. It is also seen from Fig. 4 that the BER performance improves as the number $K$ of spectrum divisions increases. This is because, as the number $K$ of subcarrier division increases, the distance between subcarriers for a STBC codeword become narrow and as consequence, the channel variation within a SFBC codeword become small (See Fig. 3).

Fig. 5 plots complementary cumulative distribution function (CCDF) of PAPR in SC-SFBC diversity. In this paper, PAPR is calculated as

$$\text{PAPR} = \max \left\{ s(n,t)^T \left[ E \left( s(n,t)^T s(n,t) \right) \right] \right\},$$

where $\{s(n,t):n=0,...,N_c-1, t=0,1,8,...,N_t-1\}$ is the 8 times over-sampled transmit signal waveform. For the comparison, PAPRs of conventional SC transmission and OFDM transmission are also plotted in Fig. 5. It is seen from Fig. 5 that PAPR increases as the number of spectrum divisions increases. However, as the number of spectrum divisions further increases, PAPR decreases. The reason for this is explain as follows. When the number of spectrum divisions is relatively small, the phase rotation among subcarriers becomes non-consecutive as the number of subcarrier division increases and as consequence, PAPR increases. However, when the number of spectrum divisions is relatively large, the distance between subcarriers for a common SC signal becomes close to be equal as the number of spectrum divisions increases (See Fig. 3). Therefore, the phase rotation among subcarriers becomes consecutive as the number of spectrum divisions increases and as consequence, PAPR decreases. It is seen from these consideration that the optimal number of spectrum divisions is $K=M$ for both BER and PAPR performances.

It is also seen from Fig. 5 that PAPR of SC-SFBC diversity is higher than that of conventional SC transmission due to SFBC encoding. However, PAPR of SC-SFBC diversity is still lower than that of OFDM transmission. For example, SC-SFBC diversity with $K=M$ can reduce the 0.1% outage PAPR, which is the PAPR value at CCDF=0.1%, by about 1.5 dB compared to OFDM transmission.

B. Impact of time and frequency-selectivity

Fig.6 plots the BER performance when using SFBC diversity with the robust FDE as a function of the normalized maximum Doppler frequency $f_d T_B$ and the normalized rms delay spread $\tau_{rms}/T_r$. The transmit $E_b/N_0$ is set to 6dB. For comparison, the performances of SFBC diversity with the conventional FDE and STBC diversity with our previously proposed MB-FDE [13] are also plotted in Fig. 6. It is seen from Fig. 6 that BER performance of SC-SFBC diversity improves as the normalized rms delay spread increases. However, as the normalized rms delay spread further increases, BER performance of SC-SFBC diversity degrades. The reason for this is explained as follows. In SC transmission, frequency diversity gain increases with increase of the normalized rms delay spread and hence, BER performance improves. However, in SFBC diversity, BER performance degrades as the normalized rms delay spread increases due to the interference caused by the orthogonality distortion of SFBC codeword. Therefore, there exists tradeoff relationship between frequency diversity gain and the interference caused by the orthogonality distortion of SFBC codeword. SC-SFBC diversity with the robust FDE can reduce the 0.1% outage PAPR, which is the PAPR value at CCDF=0.1%, by about 1.5 dB compared to OFDM transmission.
It is also seen from Fig. 6 that SC-SFBC diversity with the robust FDE achieves better BER performance than SC-STBC diversity with MB-FDE when $f_{DTB}=0.03$. This is because SFBC diversity can shorten the codeword length compared to STBC diversity. When $\tau_{rms}/T_s=2.0$ and the required BER=$10^{-3}$, SFBC diversity can tolerate 2 times larger Doppler frequency than STBC diversity.

![Image](image-url)

**Fig. 6.** Impact of the normalized maximum Doppler frequency and the normalized rms delay spread.

V. CONCLUSIONS

In this paper, we proposed the robust FDE for SC-SFBC diversity in a frequency-selective fading channel. It was shown by the computer simulation that the proposed robust FDE suppresses the interference caused by the orthogonality distortion of SFBC codeword and achieves better BER performance than the conventional FDE. Note that the robust FDE performs one-tap FDE only as well as the conventional FDE and therefore, the computational complexity of the robust FDE is almost the same as that of the conventional FDE while achieving better BER performance. Furthermore, the robust FDE can be applied to not only SFBC diversity with transmit FDE but also SFBC diversity with transmit FDE. Therefore, by using SFBC diversity with receive robust FDE and SFBC diversity with transmit robust FDE for uplink and downlink transmissions, respectively, both uplink and downlink BER performances can be improved while keeping mobile terminal structure simple as well as our previously proposed SC-STBC time division duplex (TDD) transmission [13-15].

In this paper, we assumed the perfect CSI. The channel estimation suitable for SC-SFBC diversity is left as our future work. Theoretical analysis of SC-SFBC diversity with the robust FDE is also an interesting future study.

REFERENCES