Single-Carrier Transmission with Frequency-Domain based Code-Division Multi-Access

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Abstract— Single-carrier (SC) transmission is a promising transmission scheme for uplink transmission because of its low peak-to-average power ratio (PAPR). Spread spectrum (SS) transmission achieves higher frequency diversity gain and hence improves the bit error rate (BER) performance. The authors have recently proposed SC with frequency-domain spread spectrum (SC-FDSS) transmission, where spreading and de-spreading are conducted in frequency domain. In this paper, we propose a new code-division multi-access (CDMA) scheme based on SC-FDSS for multi-user environment, called SC with frequency-domain CDMA (SC-FD-CDMA). Theoretical analysis on conditional BER of the uplink SC-FD-CDMA is presented. The BER analysis is confirmed by the computer simulation together with PAPR evaluation. BER and PAPR of SC-FD-CDMA are compared with direct-sequence CDMA (DS-CDMA) and multi-carrier CDMA (MC-CDMA).

Keywords— Single-carrier (SC) transmission, spread spectrum, code division multi-access (CDMA), uplink transmission

I. INTRODUCTION

Broadband wireless channel is characterized as a frequency-selective fading channel, in which inter-symbol interference (ISI) degrades the bit-error rate (BER) performance [1]. Multi-carrier transmission, such as orthogonal frequency division multiplexing (OFDM), is robust against frequency-selective fading but its high peak-to-average power ratio (PAPR) of transmit signal is the main drawback [2]. On the other hand, single-carrier (SC) transmission [3] is more attractive for uplink communication in LTE-Advanced (LTE-A) system because of lower PAPR, while the use of frequency-domain equalization (FDE) can take advantage of the channel frequency selectivity to improve the BER [4].

SC transmission can be combined with multi-access techniques which provide users’ orthogonality in different domains [5-7]. Among the various combinations of SC transmission and multi-access, SC with FDMA (called SC-FDMA) [8] and direct-sequence CDMA (DS-CDMA) with FDE [7] are very attractive for uplink transmissions. Multi-carrier CDMA (MC-CDMA) [9] can be possibly considered as a candidate due to its robustness against frequency selectivity.

In SC-FDMA, users are separated in frequency-domain, and hence there is no multi-user interference (MUI) [10]. However, since the number of available subcarriers is limited, a complicated resource allocation algorithm [11] is necessary. On the other hand, CDMA, which is adopted for the third-generation (3G) system [12], allows a user to share the same bandwidth. This implies that the resource allocation is not needed. Frequency diversity gain is also achievable by either time-domain (for DS-CDMA) or frequency-domain spreading (for MC-CDMA); however, strong MUI occurs since the orthogonality among different spreading codes is severely distorted through the multipath fading, and consequently degrades the BER performance.

Recently, we proposed SC with frequency-domain spread spectrum (SC-FDSS) [13] combined with orthogonal code multiplexing [14]. Spreading and de-spreading are conducted in frequency domain similar to multi-carrier spread spectrum (MC-SS) but with the aid of discrete Fourier transform (DFT). Performance of SC-FDSS was evaluated in [13, 14] to confirm that better BER is achieved compared to SC with time-domain spread spectrum (SC-TDSS). The theoretical results in [14] also showed that the inter-chip interference (ICI) in SC-FDSS is lower than SC-TDSS, implying that SC-FDSS is preferable to be used in strong-ICI environment. Note that the performance evaluation in [13, 14] were done only in single-user environment.

In this paper, we extend the single-user SC-FDSS and propose a novel multi-user SC-FDSS, called SC with frequency-domain CDMA (SC-FD-CDMA). A single-cell multi-user uplink transmission is considered. Performance evaluation of SC-FD-CDMA is done by computer simulation and compared to conventional DS-CDMA [7] and MC-CDMA [9] in terms of BER and PAPR assuming the same number of users and that of subcarriers. It will be shown that the uplink BER performance of SC-FD-CDMA is better than DS-CDMA as a contribution of lower interference.

The rest of this paper is organized as follows. The transceiver model and signal representations for SC-FD-CDMA are presented in Sect. II. Theoretical analysis on conditional BER is presented in Sect. III. Section IV presents the simulation results of BER and PAPR. Section V concludes the paper.

II. TRANSMISSION SYSTEM MODEL

Chip-spaced discrete-time signal representation is used throughout this paper. A single-cell single-antenna consisting of $U$ users is considered, where all $U$ users transmit the data to the base station. The number of available subcarriers is $N$. Fig. 1 illustrates the baseband transmission system models of (a) transmitter of the $u$-th user, $u=0\sim U-1$, and (b) receiver at the base station of uplink SC-FD-CDMA. Note that $U\leq SF$ when $SF$ represents the spreading factor.
A. Transmit Signal

A transmit symbol vector of the $u$-th user, which consists of $M=N_c/SF$ modulated symbols, is represented by $\mathbf{d}_u=[d_u(0),d_u(1),…,d_u(M-1)]^T$. $\mathbf{d}_u$ is firstly transformed into frequency domain by $M$-point discrete Fourier transform (DFT), where the $M$-point DFT matrix $\mathbf{F}_M$ is expressed by

$$\mathbf{F}_M = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ -\frac{j2\pi}{M} & e^{\frac{j2\pi}{M}} & \cdots & e^{\frac{j2\pi(M-1)}{M}} \\ \vdots & \vdots & & \vdots \\ -\frac{j2\pi(M-1)}{M} & \cdots & e^{\frac{j2\pi}{M}} & e^{\frac{j2\pi(M-1)}{M}} \end{bmatrix},$$  

(1)

and its Hermitian transpose $\mathbf{F}_M^H$ represents inverse operation. $\mathbf{D}_u=[\mathbf{D}_u(0),\mathbf{D}_u(1),…,\mathbf{D}_u(M-1)]^T$ is determined as a frequency-domain signal vector, which is given by

$$\mathbf{D}_u = \mathbf{F}_M \mathbf{d}_u.$$  

(2)

Next, $\mathbf{D}_u$ is spread to entire $N_c$ subcarriers by an $N_c \times M$ spreading matrix $\mathbf{C}_u$ yielding a frequency-domain spread signal $\mathbf{S}_u=\mathbf{C}_u \mathbf{D}_u$ where $\mathbf{S}_u=[\mathbf{S}_u(0),\mathbf{S}_u(1),…,\mathbf{S}_u(N_c-1)]^T$. $\mathbf{C}_u$ can be expressed by

$$\mathbf{C}_u = \begin{bmatrix} C_u(0) & & \cdots & \cdots & \cdots \\ \vdots & C_u(SF-1) & & \cdots & \cdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \cdots & C_u(0) & \cdots \\ 0 & \cdots & \cdots & \cdots & C_u(SF-1) \end{bmatrix},$$  

(3)

where $C_u(k)$ has unit-magnitude (i.e. $|C_u(k)|^2=1$).

We also take an advantage of using frequency-domain processing by introducing a simple frequency mapping technique to SC-FD-CDMA. An interleaved mapping is used in order to avoid the effect from frequency selectivity on the same frequency-domain component. Each element of the mapped signal vector $\mathbf{\tilde{S}}_u = [\mathbf{\tilde{S}}_u(0),\mathbf{\tilde{S}}_u(1),…,\mathbf{\tilde{S}}_u(N_c-1)]^T$ can be expressed by

$$\mathbf{\tilde{S}}_u(p+(q \times M)) = \mathbf{S}_u((p \times SF) + q),$$  

(4)

where $p=0…M-1$ and $q=0…SF-1$. Note that the interleaved mapping is also applicable in MC-CDMA.

Finally, $\mathbf{\tilde{S}}_u$ is transformed back into time domain by applying $N_c$-point inverse fast Fourier transform (IFFT) represented by $\mathbf{F}_N^H$. Time-domain transmit signal before adding guard interval $\mathbf{\tilde{s}}_u = [\mathbf{\tilde{s}}_u(0),\mathbf{\tilde{s}}_u(1),…,\mathbf{\tilde{s}}_u(N_c-1)]^T$ after passing through all processes in (1)-(4) is

$$\mathbf{\tilde{s}}_u = \sqrt{2P_u} \mathbf{F}_N^H \mathbf{\tilde{S}}_u,$$  

(5)

where $P_u$ is the average transmit power of the $u$-th user. The last $N_c$ chips of each transmission block are copied as cyclic prefix (CP) and inserted into guard interval placed at the beginning of each block and then the signals are transmitted. In summary, we also illustrate the transmission processing of SC-FD-CDMA by Fig. 2. Note that computational complexity of transmitter of SC-FD-CDMA is higher than MC-CDMA transmitter due to an implementation of DFT block (plus IFFT block when comparing with DS-CDMA), but similar to SC-FDMA transmitter.

B. Received Signal

The propagation channel is assumed to be a chip-space $L$-path frequency-selective block fading channel [1], where its impulse response between the $u$-th user and the base station is

$$h_u(r) = \sum_{l=0}^{L-1} h_{ul} \delta(r-\tau_{ul}),$$  

(6)

where $h_{ul}$ and $\tau_{ul}$ are complex-valued path gain and time delay of the $l$-th path and the $u$-th user, respectively. $\delta(\cdot)$ is the delta function.

From (5) and (6), A superposition of $U$ users’ signals is received at the base station antenna. The received signal vector after the CP removal $\mathbf{r}=[r(0),r(1),…,r(N_c-1)]^T$ can be represented as

$$\mathbf{r} = \sum_{u=0}^{U-1} \sqrt{2P_u} \mathbf{h}_u \mathbf{\tilde{s}}_u + \mathbf{n},$$  

(7)

Fig. 1. Transmission system model of SC-FD-CDMA.
while is obtained from (5), and is a noise vector in which element is zero-mean additive white Gaussian noise (AWGN) having the variance $2N_0/T_c$ with $T_c$ and $N_0$ being the chip duration and the one-sided noise power spectrum density, respectively. Channel response matrix $h_u$ is a circular matrix representing time-domain channel response between the $u$-th user and the base station, which is

$$h_u = \begin{bmatrix} h_{u,0} & h_{u,1} & \cdots & h_{u,1} \\ \vdots & \vdots & \ddots & \vdots \\ h_{u,1} & \cdots & h_{u,0} & 0 \\ 0 & \vdots & \cdots & h_{u,0} \end{bmatrix}. \tag{8}$$

The received signal vector $r$ is transformed into frequency domain by $N_c$-point FFT, obtaining the frequency-domain received signal $R$ as

$$R = \sum_{u=0}^{U-1} \sqrt{2P_u} F_u h_u \tilde{x}_u + F_u n_u = \sum_{u=0}^{U-1} \sqrt{2P_u} F_u h_u F_u^T \tilde{S}_u + F_u n_u, \tag{9}$$

where the frequency-domain channel response $H_u$ is

$$H_u = F_u h_u F_u^T = \text{diag}[H_{u,0}(0),\ldots,H_{u,N_c}(N_c-1)].$$

FDE based on minimum mean-square error criterion (MMSE-FDE) is introduced for mitigating the ISI occurred by frequency-selective fading channel. The frequency-domain received signal after applying MMSE-FDE of the $u$-th user is

$$\hat{R}_u = W_u R,$$ \tag{10}

where $W_u = \text{diag}(W_u(0),\ldots,W_u(N_c-1))$ is a $N_c \times N_c$ diagonal matrix. In this paper, a conventional MMSE-FDE, which minimize the mean-square error (MSE) between $\tilde{x}_u$ and $\hat{R}_u$, is considered for a fair comparison with DS-CDMA and MC-CDMA. $W_u(k)$ is described in [15] as

$$W_u(k) = \frac{P_u T_c}{N_0} H_u(k) \frac{1}{\sum_{\nu=0}^{N_0-1} |H_u(\nu)|^2 + 1}.$$

After that, de-mapping is applied to the received signal after applying MMSE-FDE $\hat{R}_u$, obtaining the frequency-domain signal $\tilde{R}_u = [\tilde{R}_u(0), \tilde{R}_u(1), \ldots, \tilde{R}_u(N_c-1)]^T$. De-mapping can be expressed by

$$\tilde{R}_u((p \times SF) + q) = \tilde{R}_u(p + (q \times M)),$$ \tag{12}

where $p=0-M-1$ and $q=0-SF-1$. It can be seen that de-mapping in (13) is simply an inverse operation of (4).

De-spreading is also applied in frequency-domain approach by simply multiplying $\tilde{R}_u$ by an inverse operation of (3), resulting in frequency-domain vector $\tilde{D}_u = [\tilde{D}_u(0), \tilde{D}_u(1), \ldots, \tilde{D}_u(M-1)]^T$ as

$$\tilde{D}_u = F_u^H \tilde{D}_u. \tag{14}$$

III. BER ANALYSIS

In this section, conditional SINR and BER analysis is derived for the proposed SC-FD-CDMA uplink. SINR is derived by firstly determining a frequency-domain signal after FDE of the $j$-th user $\tilde{R}_j = [\tilde{R}_j(0), \tilde{R}_j(1), \ldots, \tilde{R}_j(N_c-1)]^T$ as

$$\tilde{R}_j(k) = \sqrt{2P_j} \tilde{H}_{j,j}(k) C_j(k) D_j \left( \frac{k}{SF} \right) + \sum_{u=0}^{U-1} \sqrt{2P_u} \tilde{H}_{u,j}(k) C_u(k) D_u \left( \frac{k}{SF} \right) + \tilde{W}_j(k) N(k).$$ \tag{15}

Here, $\tilde{H}_{u,j}(k)$, $\tilde{H}_j(k)$ and $\tilde{W}_j(k)$ are expressed as follows,

$$\begin{aligned}
\tilde{H}_{u,j}(k) &= \tilde{H}_j(k) \tilde{W}_j(k) \\
\tilde{H}_j((p \times SF) + q) &= H_j((p + (q \times M))) \\
\tilde{W}_j((p \times SF) + q) &= W_j((p + (q \times M))
\end{aligned} \tag{16}$$

where $p=0-M-1$ and $q=0-SF-1$. Substitute (15) into (13) yields

$$\tilde{D}_j(m) = \sum_{k=m(SF-1)}^{(m+1)(SF-1)-1} \sqrt{2P_j} \tilde{H}_{j,j}(k) C_j(k) C_j(k) D_j(m) + \sum_{k=m(SF-1)}^{(m+1)(SF-1)-1} \sum_{u=0}^{U-1} \sqrt{2P_u} \tilde{H}_{u,j}(k) C_u(k) C_u(k) D_u(m), \tag{17}$$

$$+ \sum_{k=m(SF-1)}^{(m+1)(SF-1)-1} \tilde{W}_j(k) N(k).$$
where \( m = 0 \sim N_c/SF - 1 \). It is observed from (17) that the second term and the third term represent residual MUI \( \mu_m \) and noise \( \mu_{N_c} \), respectively. The variance of \( \mu_{MUI} \) is expressed by

\[
2\sigma^2_{MUI} = E[|\mu_{MUI}|^2] = \sum_{k=1}^{U-1} \sum_{j=0}^{SF-1} \left( \frac{1}{SF} \sum_{n=-\infty}^{\infty} H_{j,H_k}(k)^2 \right),
\]

which is similar to MUI in MC-CDMA [16].

However, it is also observed that an additional residual ICI occurs when the frequency-domain component \( \tilde{D}_h(m) \) in (17) is transformed back into time-domain received symbol. The variance of additional residual ICI is expressed by

\[
2\sigma^2_{ICI} = P_j \left( \frac{1}{N_c/SF} \sum_{q=0}^{N_c/SF-1} \left( \frac{1}{SF} \sum_{j=0}^{SF-1} |\tilde{H}_{j,H_k}(q)|^2 \right) \right),
\]

where \( P_j \), \( q=0 \sim N_c/SF - 1 \) represents an equivalent channel gain after de-spreading of the \( j \)-th user, which is

\[
\tilde{H}_{j,H_k}(q) = \sum_{k=qSF}^{qSF-1} \tilde{H}_{j,H_k}(k). \tag{20}
\]

Finally, the conditional SINR for the \( j \)-th user with the given \( P_j \) and \( \{H_u\} u=0 \sim U-1 \) is expressed by

\[
\gamma(P_j,\{H_u\}) = \frac{2P_j}{N_c/SF} \sum_{q=0}^{N_c/SF-1} \left( \frac{1}{2\sigma^2_{ICI} + 2\sigma^2_{MUI} + \sigma^2_{N_c}} \right), \tag{21}
\]

The conditional BER of the \( j \)-th user assuming QPSK modulation is given as

\[
p_b(P_j,\{H_u\}) = 0.5 \times erfc\left(\frac{0.25\gamma(P_j,\{H_u\})}{\sqrt{2}}\right), \tag{22}
\]

where \( erfc(\cdot) \) is complementary error function. The theoretical average BER is numerically computed by averaging (22) over all possible \( \{H_u\} \). The theoretical BER performance is shown in Section IV together with simulation results.

### IV. PERFORMANCE EVALUATION

Numerical and simulation parameters are summarized in Table 1. We assume a single-cell with single-antenna base station and \( U \) multi-user uplink transmission environment. A frequency-selective fading channel having chip-spaced \( L=16 \) path uniform power delay profile is assumed. 4095-bit long pseudo noise (PN) sequence is used as spreading code \( \{C_{u}(0),\ldots,C_{u}(SF-1)\} \) for the \( u \)-th user.

#### A. PAPR Performance

PAPR over a block of transmission [17] is defined as

\[
\text{PAPR} = \max \{\|\tilde{s}(n)\|^2 : n = 0, 1, \ldots, N_c - 1\} / \mathbb{E}[\|\tilde{s}(n)\|^2],
\]

where \( V \) represents oversampling factor. We use complementary cumulative distribution function (CCDF) as an indicator of PAPR performance. Note that Nyquist pulse shaping with roll-off factor \( \alpha=0 \) (i.e., frequency-domain ideal rectangular filter) is considered in this paper.
Fig. 3 shows the CCDF of PAPR of single-user transmit signal in DS-CDMA, MC-CDMA, and the proposed SC-FD-CDMA, where the PAPR is evaluated at $SF=16$. DS-CDMA provides the lowest transmit PAPR among these transmission schemes. PAPR of the proposed SC-FD-CDMA is higher than DS-CDMA, where PAPR at 0.1% outage probability (PAPR@0.1%) is approximately 0.9 dB higher than DS-CDMA. This is because frequency interleaving technique in the proposed SC-FD-CDMA decreases correlation among frequency-domain components. However, PAPR@0.1% of the proposed SC-FD-CDMA is 1.3 dB lower than MC-CDMA slow transmit power control (TPC) (i.e., frequency-domain components. However, PAPR@0.1% of the proposed SC-FD-CDMA decreases correlation among frequency-domain components. However, PAPR@0.1% of the proposed SC-FD-CDMA is 1.3 dB lower than MC-CDMA since the waveform of SC-FD-CDMA remains SC waveform property.

B. BER Performance

Fig. 4 shows the uplink BER performance as a function of average received bit energy-to-noise power spectrum density ratio $E_b/N_0=0.5(P_u/Tc/N_0)(SF)(1+N_c/N_s)$ when $SF=16$. Ideal slow transmit power control (TPC) (i.e., $P_u=P$ for all $u$) is assumed. BER of SC-FD-CDMA is compared with DS-CDMA and MC-CDMA at the same number of users $U$. Also plotted in Fig. 4 are the theoretical BER performance curves obtained using the conditional BER expression derived in Sect. III.

It can be seen from Fig. 4 that the BER performance of SC-FD-CDMA provides better BER than DS-CDMA in every $U$. The reasons are well described in [13] as the SC-FD-CDMA can achieve more ICI mitigation inherited from frequency interleaving in (4), and lower phase error after de-spreading since the de-spreading is done in frequency domain. The BER of the proposed SC-FD-CDMA is slightly worse than MC-CDMA due to the additional residual ICI. However, SC-FD-CDMA produces lower PAPR of the transmit signal waveform. In addition, a fairly good agreement is observed between the theoretical and simulation results.

V. CONCLUSION

In this paper, SC-FDSS with orthogonal code multiplexing was extended to multi-user uplink transmission called SC-FD-CDMA. In SC-FD-CDMA, spreading and de-spreading are done in frequency-domain, providing additional frequency-diversity gain and robustness against ISI and MUI. Simulation results assuming the single-cell environment confirmed that the proposed SC-FD-CDMA improves the BER performance compared to DS-CDMA. It was also clarified that the proposed SC-FD-CDMA provides a similar BER performance with lower transmit PAPR compared to MC-CDMA.

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