Frequency-Domain Single-Carrier Spread Spectrum with Joint FDE and Spectrum Combining

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Abstract—In this paper, a DFT-based frequency-domain spread spectrum (FDSS) technique for SC transmission is introduced. The transmitted data is initially transformed to frequency-domain signal by using DFT. Frequency-domain spreading can achieve large frequency diversity gain through joint frequency-domain equalization (FDE) and spectrum combining. Performance of the proposed FDSS is evaluated by computer simulation in aspects of bit-error rate (BER), peak-to-average power ratio (PAPR), and average throughput, and then compared to the conventional TDSS scheme with spreading sequence.

Keywords—Single-carrier (SC) transmission; spread spectrum (SS); frequency-domain equalization (FDE)

I. INTRODUCTION

Broadband wireless channel is characterized as a frequency-selective fading channel, in which the inter-symbol interference (ISI) degrades system performance in terms of bit-error rate (BER) [1]. Orthogonal frequency division multiplexing (OFDM) is robust against frequency-selective fading, but its high peak-to-average power ratio (PAPR) property is the main drawback [2]. On the other hand, single-carrier (SC) transmission [3] has lower PAPR than OFDM, while the use of minimum mean-square error based frequency-domain equalization (MMSE-FDE) [4] can mitigate the impact of ISI. Hence, SC with FDE (SC-FDE) is more attractive for uplink transmission.

SC signal can be generated by inserting discrete Fourier transform (DFT) in OFDM transmitter [5], which can introduce the frequency-domain signal processing at the transmitter. In frequency-domain processing, transmit filtering can be simply done as one-tap multiplication [6], whereas the flexibility of signal manipulation is further achieved.

To take advantage of using frequency-domain signal processing, in this paper, we introduce a DFT-based frequency-domain spread spectrum (SS) for SC-FDE (hereinafter, FDSS-SC-FDE). In the proposed FDSS-SC-FDE, data block to be transmitted is firstly transformed into frequency-domain signal by DFT. Multiple copies of the frequency-domain signal are mapped over the available frequency bandwidth (which is equivalent to spectrum spreading) and any shape of filtering is applied. Fractional spreading is available where it is not in conventional time-domain SS (TDSS). By using the FDSS, signal bandwidth can be freely chosen in order to obtain more frequency diversity gain, indicating that spreading factor can be above the limit of transmission using square-root Nyquist transmit filtering [7]. Although the proposed FDSS-SC-FDE is similar to multi-carrier SS (MC-SS) [8], the resulting signal waveform retains the property of SC signal, and consequently, has lower PAPR compared to MC-SS signal. Joint MMSE-FDE and spectrum combining [9] is served at the receiver as de-spreading technique, which provides additional diversity gain with much simpler implementation compared to TDSS receiver with the same performance [10].

Since it is known that any modifications on transmit signal processing in SC-FDE leads to changes in system performance [11], performance of the proposed FDSS-SC-FDE should be evaluated in aspects of both PAPR and BER, and compared with the TDSS transmission [12, 13] at the same transmission bandwidth. It has also been discussed in the previous literature that the use of spreading technique decreases maximum throughput, average throughput of the proposed FDSS-SC-FDE should be examined. We provide the conditional BER analysis for the proposed scheme, which is further used to calculate theoretical average BER by Monte-Carlo numerical computational method, and finally the average BER is compared with the computer simulation.

The rest of this paper is organized as follows. System model of FDSS-SC-FDE is introduced in Section II. Theoretical analysis on signal-to-interference plus noise power ratio (SINR) and conditional BER is shown in Section III. Section IV shows the performance evaluation, and Section V concludes the paper.

II. TRANSMISSION SYSTEM MODEL

We assume single-user block transmission of data-modulated symbols over available $N_c$ subcarriers, implying that the spreading factor is $SF=N_c/M$. In the proposed transmission scheme, DFT and its inverse operation are used. Joint MMSE-FDE with spectrum combining [9] is served at the receiver. The transceiver of SC transmission using the proposed FDSS-SC-FDE is illustrated by Fig. 1(a). We also depict the transceiver of conventional TDSS for comparison by Fig. 1(b) (referred by [6]).

A. Transmitter

Firstly, a block of $M$ data-modulated symbol $d=[d(0), d(1), \ldots, d(M-1)]^T$ is transformed into frequency domain by $M$-point DFT. The frequency-domain original signal
vector $\mathbf{D}=[D(0),D(1),...,D(M-1)]^T$ is $\mathbf{D}=\mathbf{F}_M\mathbf{d}$, where $\mathbf{F}_M$ is $M$-point DFT matrix given by

$$
\mathbf{F}_M = \frac{1}{\sqrt{M}} \begin{bmatrix}
1 & e^{-j2\pi(1)M} & \cdots & e^{-j2\pi(M-1)M} \\
1 & e^{-j2\pi(1)M} & \cdots & e^{-j2\pi(M-1)M} \\
\vdots & \vdots & \ddots & \vdots \\
1 & e^{-j2\pi(M-1)M} & \cdots & e^{-j2\pi(M-1)M}
\end{bmatrix}.
$$

(1)

Next, $\mathbf{D}$ is spread over available $N_c$ subcarriers. We introduce a spreading matrix $\mathbf{P}$ with the dimension of $N_c\times M$. For fair comparison with TDSS, equal transmit power allocation is considered with power normalization factor $1/\sqrt{SF}$. Spreading matrix $\mathbf{P}$ is expressed by

$$
\mathbf{P} = \frac{1}{\sqrt{SF}} \begin{bmatrix}
\mathbf{I}_{M,0} & \\
\mathbf{I}_{M,1} & \\
\vdots & \\
\mathbf{I}_{M, SF-1} &
\end{bmatrix}_{N_c\times M},
$$

(2)

where $\mathbf{I}_{M,j}$ is an $M\times M$ identity matrix. In case of fractional spreading where $SF$ is not integer, $\mathbf{I}_{M, SF-1}$ is rewritten as follow.

$$
\mathbf{I}_{M, SF-1} = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 1 & 0
\end{bmatrix}_{(SF-1)M \times M}.
$$

(3)

After that, $N_c$-point inverse fast Fourier transform (IFFT) is applied for transforming the frequency-domain spread signal back to time-domain signal. Time-domain transmit signal $s=[s(0),s(1),...,s(N_c-1)]^T$ after passing through all processes in (1) and (2) can be expressed as

$$
s = \mathbf{F}_{N_c}^H \mathbf{PD} = \mathbf{F}_{N_c}^H \mathbf{P} \mathbf{F}_M \mathbf{d}.
$$

(4)

Finally, the last $N_c$ samples of transmit block are copied as a cyclic prefix (CP) and inserted into the guard interval (GI), then a CP-inserted signal block of $N_c+N_c$ samples is transmitted.

**B. Receiver**

The transmission is conducted under independent $L$-path block Rayleigh fading channel [1]. The channel response is

$$
h_{\tau}(\tau) = \sum_{l=0}^{L-1} h_l \delta(\tau - \tau_l),
$$

(5)

where $h_l$ and $\tau_l$ are complex-valued path gain and time delay of the $l$-th path, respectively. $\delta(\cdot)$ is the delta function. The received signal after CP removal, $r=[r(0),r(1),...,r(N_c-1)]^T$, is

$$
r = \sqrt{2E_s/T_s} h \mathbf{s} + \mathbf{n},
$$

(6)

where $E_s$ and $T_s$ are symbol energy and symbol duration, respectively. Transmitted signal $\mathbf{s}$ is obtained from (5), and $\mathbf{n}$ is noise vector in which element is zero-mean additive white Gaussian noise (AWGN) having the variance $2N_0/T_s$ with $N_0$ being the one-sided noise power spectrum density. In addition, channel response matrix $\mathbf{h}$ is a circular matrix representing time-domain channel impulse response, which is

$$
\mathbf{h} = \begin{bmatrix}
h_0 & h_{L-1} & \cdots & h_1 \\
h_1 & h_0 & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
h_L-1 & h_1 & \cdots & h_0
\end{bmatrix}.
$$

(7)

$r$ is then transformed into frequency domain by $N_c$-point FFT, obtaining the frequency-domain received signal $\mathbf{R}$ as

$$
\mathbf{R} = \sqrt{2E_s/T_s} \mathbf{h} \mathbf{s} + \mathbf{F}_{N_c} \mathbf{n} = \sqrt{2E_s/T_s} \mathbf{F}_{N_c}^H \mathbf{PD} \mathbf{s} + \mathbf{F}_{N_c} \mathbf{n}.
$$

(8)

Here, the frequency-domain channel response is

$$
\mathbf{F}_{N_c} \mathbf{h}^T = \text{diag}[H_s(0),...,H_s(N_c-1)] = \mathbf{H}_s.
$$

(9)

Joint MMSE-FDE and spectrum combining is applied at the receiver to reduce the impact from frequency-selective fading and to recover the original $M$ frequency-domain signal. Joint MMSE-FDE with spectrum combining is introduced by $\mathbf{W}$ with the dimension of $M\times N_c$, which is

$$
\mathbf{W} = [\mathbf{W}_0 \mathbf{W}_1 \cdots \mathbf{W}_{SF-1}]_{M\times N_c},
$$

(10)

where $\mathbf{W}_k$ is an $M\times M$ matrix expressed by

$$
\mathbf{W}_k = \text{diag} \left[ \frac{W(nSF/M)}{SF}, \frac{W((n+1)SF/M)}{SF}, \ldots \right],
$$

(11)

Note that in case of fractional spreading when $SF$ is not integer, the dimension of $\mathbf{W}_{SF-1}$ is reduced to be $(\lfloor SF \rfloor M) \times M$.

By using MMSE criterion, the FDE weight with respect to each frequency index $W(k), k=0-N_c-1$ is calculated in the same pattern as in [10], which is

$$
W(k) = \frac{(1/\sqrt{SF})H'_s(k)}{\sum_{g=0}^{M-1} H'_s(k \mod M + gM)^2 + (E_s/N_0)^{-1}},
$$

(12)
where $H_t(k)$ is the elements in $H_t$ with respect to each frequency index. The frequency-domain received signal after equalization and spectrum combining $\hat{d} = WR$ is finally transformed back into time-domain by $M$-point inverse DFT (IDFT). Therefore, the time-domain received signal after equalization and combining $\hat{d} = [\hat{d}(0), \hat{d}(1), \ldots, \hat{d}(M - 1)]^T$ is

$$\hat{d} = F^H_D \hat{\mathbf{d}} = F^H_D W R .$$

Spectrum combining can be illustrated as shown in Fig. 2. It can be observed that frequency diversity gain is obtained as a contribution of spreading at the transmitter, where the same frequency spectrum is sent in different frequency index.

### III. BER Analysis

In this section, conditional SINR and BER analysis is derived for the proposed FDSS-SC-FDE using joint FDE and spectrum combining. SINR is derived by firstly beginning from the received symbol $\hat{d}(m)$.

$$\hat{d}(m) = \sqrt{2E_s/T_c} \left( \frac{1}{M} \sum_{q=0}^{M-1} \tilde{H}(q) \right) d(m)$$

$$+ \sqrt{2E_s/T_c} \left( \frac{1}{M} \sum_{q=0}^{M-1} \tilde{H}(q) \sum_{r=1}^{\frac{M}{2}} d(r) \exp(j2\pi q t - \tau/r) \right),$$

$$+ \frac{1}{M} \sum_{q=0}^{M-1} \tilde{N}(q) \exp(j2\pi q t/M)$$

where $\tilde{H}(q)$ and $\tilde{N}(q)$ are expressed as follows.

From (15), it can be observed that $\tilde{H}(q)$ is equivalent channel after joint FDE and spectrum combining. The second term and the third term in (14) represent residual ISI and noise, respectively. It can be observed from (14) that $\tilde{d}(m)$ is a complex-valued random variable with the mean value of $\sqrt{2E_s/T_c} \left( \frac{1}{M} \sum_{q=0}^{M-1} \tilde{H}(q) \right)$.

Hence, the conditional SINR for the given $E_s/N_0$ and $H_t$ is expressed by

$$\gamma_{FD} \left( \frac{E_s}{N_0}, H_t \right) = \frac{2E_s \left( \frac{1}{M} \sum_{q=0}^{M-1} \tilde{H}(q) \right)^2}{\frac{1}{M} \sum_{q=0}^{M-1} \tilde{N}(q)^2}.$$  (17)

For simplicity of analysis, the residual ISI plus noise after FDE is assumed to be a zero-mean complex-valued Gaussian random variable [13]. The conditional BER when QPSK modulation is used is given as

$$p_b \left( \frac{E_s}{N_0}, H_t \right) = 0.5 \times \text{erfc} \left( \sqrt{0.25 \times \gamma \left( \frac{E_s}{N_0}, H_t \right)} \right),$$  (18)

where $\text{erfc}(\cdot)$ is complementary error function. The theoretical average BER is numerically computed by averaging (18) over all possible $H_t$. The theoretical BER performance is shown in Section IV together with simulation results.

As a comparison, we also consider the TDSS using spreading sequence [12]. We can see that (17) and SINR indicated in [12] are different, but still difficult to exactly determine that which one is better. To clarify the difference between them, theoretical and simulated BER performance is shown in the next section.

### IV. Performance Evaluation

Numerical and simulation parameters are summarized in Table I. We assume QPSK block transmission with the number
of available subcarriers $N_c = 256$. System performance is evaluated in terms of BER, PAPR, and average throughput, while the BER is also compared with the theoretical BER derived in Sect. III.

A. BER Performance

Fig. 3 shows the comparison of BER performance of SC transmission with proposed DFT-based FDSS-SC-FDE and conventional TDSS transmission as a function of average received bit energy-to-noise power spectrum density ratio $E_b/N_0 = 0.5(E_s/N_0)(1 + N_c/N_s)$. It is seen that BER performance of the proposed FDSS transmission outperforms the conventional TDSS transmission using MMSE-FDE. This is because joint MMSE-FDE with spectrum combining minimizes the mean-square error (MSE) between the signals before spreading and after de-spreading. The TDSS receiver achieves the same BER performance as joint MMSE-FDE at the cost of higher complexity due to the requirement of matrix inversion [10]. In addition, we can observe a fairly good agreement between the theoretical and simulated results.

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As discussed earlier, the proposed FDSS-SC-FDE is compatible with fractional spreading, particularly spreading with non-integer $SF$. Fig. 4 shows the comparison of BER performance as a function of $SF$ where the average received $E_b/N_0$ is 10 dB. For example, we evaluate the BER performance at $SF = 1.33, 2.66, 5.33, \text{and} 10.66$, which are equivalent to $M=192, 96, 48, \text{and} 24$, respectively. This implies that we can reduce the unnecessary bandwidth expansion when arbitrary $SF$ is available.

B. PAPR Performance

PAPR over a block transmission is defined as

$$PAPR = \max_{n=0}^{N_c-1} \left\{ \frac{\max |s(n)|^2}{E[|s(n)|^2]} \right\},$$

(19)

where $O.S.$ represents oversampling factor.

We use the complementary cumulative distribution function (CCDF) as an indicator of PAPR performance. Fig. 5 shows the CCDF of PAPR of both transmission scheme. We also evaluate the PAPR performance of arbitrary $SF = 1.6, 3.2, \text{and} 6.4$ (these are selected in order to make $M$ be integer and to be between $SF = 2, 4, \text{and} 8$, respectively). PAPR slightly increases when $SF$ increases in TDSS transmission. However, it is noticed that PAPR reduces when $SF = 2$ in the proposed FDSS, and then turns to increase again when $SF = 4$. This is because of the interleave property in time-domain signal of the proposed FDSS, which decreases the peak occurred from pulse overlapping when $SF < 4$ [14]. However, PAPR turns to increase when the interval between each time-domain signal is excessively wide.

We can observe that the transmission using proposed FDSS-SC-FDE with $SF = 2$ is very attractive since it provides significant BER improvement with low PAPR, especially the flexibility of signal manipulation is achieved when the proposed FDSS approach is used.

C. Throughput Performance

In this paper, we assume 512-symbol QPSK-modulated packet transmission. The number of required transmission blocks per packet increases as $SF$ increases. Here, channel coding is not considered. The average throughput $\eta$ (in bps/Hz) is defined as [15]

$$\eta = 2 \times (1 - \text{PER}) \times (SF)^{-1} \times (1 + N_g/N_c)^{-1},$$

(20)

where $\text{PER}$ is the packet error rate. The throughput performance computed from the measured $\text{PER}$ is shown in Fig. 6. We evaluate the throughput performance for $SF = 1, 2, \text{and} 4$ for both TDSS and proposed FDSS. It can be seen from the figure that throughput performance of the proposed FDSS outperforms the TDSS in every $SF$ as a merit from BER improvement. We can also observe that increasing $SF$ can improve the throughput performance when the average received $E_b/N_0$ is low. However, at high $E_b/N_0$ region, increasing $SF$ decreases the maximum throughput instead.

![Figure 3. BER performance of SC with various spreading techniques.](image-url)
The above discussion implies that the proposed FDSS works effectively when the received signal power is low. However, excessive SF may not provide improvement, as it can be observed that there exists not much improvement when the SF increases from 2 to 4.

V. CONCLUSION

In this paper, we proposed an FDSS-SC-FDE using joint FDE and spectrum combining. Transmit data is firstly transformed to frequency-domain and then mapping is done over the available frequency bandwidth, which is equivalent to spreading. Both theoretical analysis on the conditional BER and simulation results confirmed that significant BER and throughput improvements at the low-received-power region are obtained compared to both conventional TDSS transmission and transmission without spreading. The proposed scheme also provides the lowest PAPR when spreading factor SF = 2.

REFERENCES