Decision Feedback Channel Estimation for OFDM with STTD

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Abstract—In this paper, we propose decision feedback channel estimation (DFCE) for orthogonal frequency division multiplexing (OFDM) with space-time coded transmit diversity (STTD). Two transmit channels are simultaneously estimated by transmitting the STTD encoded pilot. To improve the tracking ability of the channel estimation against fast fading, decision feedback is also used in addition to pilot. For noise reduction and prevention of error propagation, the cascade of two averaging filters in time- and frequency-domain is applied. The average bit error rate (BER) performance of OFDM with STTD is evaluated by computer simulation. It is found that the use of DFCE can achieve a degradation in the $E_b/N_0$ from ideal CE of as small as 2dB for an average BER=10^{-3}. With receive antenna diversity reception, $E_b/N_0$ degradation can be further reduced to as small as 1dB.

Keywords—OFDM, STTD, channel estimation, decision feedback

I. INTRODUCTION

In the next generation mobile communication systems, very high-speed and high-quality data transmission will be required. However, the transmission performance is severely degraded due to severe inter-symbol interference (ISI) resulting from frequency-selective fading. Recently, orthogonal frequency division multiplexing (OFDM) [1, 2] has been attracting much attention. OFDM uses a number of lower rate subcarriers to prevent the ISI and to efficiently utilize the limited frequency band. However, its transmission performance is subjected to frequency-nonselective fading and the bit error rate (BER) performance is severely degraded compared to no fading. To improve the performance in frequency-nonselective fading, antenna diversity technique is attractive [3]. Recently, transmit diversity is attracting attention because it can reduce the number of diversity antennas at a mobile receiver and can alleviate the complexity problem of mobile terminal [4]. Especially, Alamouti’s space-time coded transmit diversity (STTD) [5] has been gaining a lot of attention for downlink (base-to-mobile) transmissions [6, 7].

For STTD decoding, channel estimation (CE) is necessary. The pilot-assisted CE is well-known, where CE is carried out using the periodically transmitted known pilot symbols [8, 9]. As the pilot transmission rate increases, more accurate channel estimation is possible, but the power loss due to pilot insertion increases. In this paper, to avoid this power loss problem, we propose a pilot-assisted decision feedback CE (PA-DFCE) for OFDM with STTD. In PA-DFCE, STTD encoded pilot block is transmitted periodically for simultaneous estimation of the two transmit channels. If pilot is not available, the previous decision symbol is fed back and used as the pilot. To reduce the noise effect in CE, we apply the cascade of two averaging filters in time- and frequency-domain.

The remainder of this paper is organized as follows. The transmission system model of OFDM with STTD is presented in Sect. 2. The proposed PA-DFCE is described in Sect. 3. In Sect. 4, the computer simulation results for the BER performance of OFDM with STTD using the proposed PA-DFCE are presented and compared with CE without DF and with differential STTD [10] which requires no CE. The paper is concluded in Sect. 5.

II. TRANSMISSION SYSTEM MODEL OF OFDM WITH STTD

Figure 1 shows the transmitter/receiver structure. We consider OFDM having $K$ orthogonal subcarriers. Throughout the paper, $T_s$-spaced discrete-time representation for the OFDM signal is used, where $T_s$ represents the sampling period for fast Fourier transform (FFT). Without loss of generality, we consider the transmission of $2K$ data-modulated symbols during two consecutive OFDM signaling intervals, i.e., $0 \leq t < 2(K+N_g^0)$, where $N_g^0$ is the guard interval (GI).

A. STTD Encoding

Figure 1(a) shows the transmitter structure for STTD encoding in the time-domain. At the transmitter, the data-modulated symbol sequence is divided into a sequence of blocks, each with $K$ symbols. STTD encoding in the time-domain can be implemented as in [11-13], that does not require subcarrier-by-subcarrier STTD encoding. The OFDM signals, $\{s_{e,n}(t); t=0 \sim K-1\}$ and $\{s_{o,n}(t); t=0 \sim K-1\}$, of even and odd blocks before STTD encoding are generated by the $K$-point inverse FFT (IFFT) as

$$s_{e,n}(t) = \sqrt{S} \sum_{k=0}^{K-1} d_{e,n}(k) \exp(j2\pi nt/K),$$
$$s_{o,n}(t) = \sqrt{S} \sum_{k=0}^{K-1} d_{o,n}(k) \exp(j2\pi nt/K),$$

where $\{d_{e,n}(k); k=0 \sim K-1\}$ and $\{d_{o,n}(k); k=0 \sim K-1\}$ are respectively the data-modulated symbol sequences of the $n$th even block and odd block and $S$ is the total transmit power per subcarrier. The $n$th even and odd blocks of STTD encoded OFDM signal to be transmitted from the $0$th antenna are $\{s_{e,n}(t); t=0 \sim K-1\}$ and $\{-s_{o,n}^*(K-t); t=0 \sim K-1\}$, respectively, and those to be transmitted from the $1$st antenna are $\{s_{o,n}(t); t=0 \sim K-1\}$ and $\{-s_{e,n}^*(K-t); t=0 \sim K-1\}$, respectively. After insertion of an $N_t$-sample GI, the STTD encoded OFDM signals are transmitted from two antennas.
point FFT is applied to decompose the received signal into single sided power spectrum density. After removal of GI, equivalent low-pass representation as transmit antennas are received by frequency-selective channel. The propagation channel is assumed to be a $m$th receive antenna are denoted as $\{s_{m,n}(t)\}$ and $\{r_{m,n}(t)\}$, respectively. They are expressed using the equivalent low-pass representation as

$$
\begin{align*}
\{s_{m,n}(t)\} &= \sum_{l=0}^{L-1} \{e_{m,n}(t-l\tau) + \tilde{e}_{m,n}(t-l\tau)\} \\
&+ \{\eta_{m,n}(t)\} \\
\{r_{m,n}(t)\} &= \sum_{l=0}^{L-1} \{e_{m,n}(t-l\tau) - \tilde{e}_{m,n}(t-l\tau)\} \\
&+ \{\eta_{m,n}(t)\}
\end{align*}
$$

for $r=N_c-1$, where $\{\eta_{m,n}(t)\}$ and $\{\tilde{\eta}_{m,n}(t)\}$ represent the independent additive white Gaussian noise (AWGN) processes having zero mean and variance $2N_c/T_c$ with $N_c$ representing the single sided power spectrum density. After removal of GI, K-point FFT is applied to decompose the received signal into K subcarrier components. The subcarrier components, $\{\tilde{a}_{m,n}(t)\}$ and $\{\tilde{b}_{m,n}(t)\}$, of the $m$th even and odd blocks are given by

where $\tilde{H}_{0,0}(m,k)$ and $\tilde{H}_{1,0}(m,k)$ respectively represent the channel gain and noise due to AWGN at the $k$th subcarrier and are given by

$$
\begin{align*}
\tilde{H}_{0,0}(m,k) &= \frac{1}{K} \sum_{l=0}^{K-1} \{e_{m,n}(t-l\tau)\} \exp(-j2\pi t\tau / K) \\\n&+ \Pi_{0,0}^{(m)}(k) \\
\tilde{H}_{1,0}(m,k) &= \frac{1}{K} \sum_{l=0}^{K-1} \{\tilde{e}_{m,n}(t-l\tau)\} \exp(-j2\pi t\tau / K) \\\n&+ \Pi_{1,0}^{(m)}(k)
\end{align*}
$$

Subcarrier-by-subcarrier STTD decoding is carried out as follows [5]:

$$
\begin{align*}
\tilde{d}_{c,n}(k) &= \frac{1}{N_c-1} \sum_{m=0}^{N_c-1} \{\tilde{H}_{0,0}(m,k)R_{c,n}^{(m)}(k) + \tilde{H}_{1,0}^{(m)}(k)R_{c,n}^{(m)*}(k)\} \\
&+ \Pi_{0,0}^{(m)}(k) \\
\tilde{d}_{o,n}(k) &= \frac{1}{N_c-1} \sum_{m=0}^{N_c-1} \{\tilde{H}_{1,0}^{(m)*}(k)R_{c,n}^{(m)}(k) - \tilde{H}_{0,0}^{(m)}(k)R_{c,n}^{(m)*}(k)\} \\
&+ \Pi_{1,0}^{(m)}(k)
\end{align*}
$$

where $\tilde{d}_{c,n}(k)$ and $\tilde{d}_{o,n}(k)$ are the soft decision values and $\tilde{H}_{0,0}^{(m)}(k)$ and $\tilde{H}_{1,0}^{(m)}(k)$ are the channel estimates of $\sqrt{S}H_{0,0}^{(m)}(k)$ and $\sqrt{S}H_{1,0}^{(m)}(k)$, respectively.

If we assume ideal channel estimation (i.e., $\tilde{H}_{0,0}^{(m)}(k)=\sqrt{S}H_{0,0}^{(m)}(k)$), substituting Eq. (3) into Eq. (5) gives

$$
\begin{align*}
\tilde{d}_{c,n}(k) &= d_{c,n}(k) \sum_{m=0}^{N_c-1} \left[ \sqrt{S}H_{0,0}^{(m)}(k) + \sqrt{S}H_{1,0}^{(m)}(k) \right]^{2} \\
&+ \sum_{m=0}^{N_c-1} \left[ \sqrt{S}H_{0,0}^{(m)*}(k)R_{c,n}^{(m)}(k) + \sqrt{S}H_{1,0}^{(m)*}(k)\Pi_{0,0}^{(m)}(k) \right]^{2} \\
\tilde{d}_{o,n}(k) &= d_{o,n}(k) \sum_{m=0}^{N_c-1} \left[ \sqrt{S}H_{1,0}^{(m)*}(k)R_{c,n}^{(m)}(k) - \sqrt{S}H_{0,0}^{(m)}(k)\Pi_{0,0}^{(m)}(k) \right]^{2} \\
&+ \sum_{m=0}^{N_c-1} \left[ \sqrt{S}H_{1,0}^{(m)*}(k)R_{c,n}^{(m)*}(k) + \sqrt{S}H_{0,0}^{(m)}(k)\Pi_{0,0}^{(m)*}(k) \right]^{2}
\end{align*}
$$

where the first term is the desired signal component and the second the noise component. Eq. (6) shows that STTD can achieve the diversity gain of 2-branch receive antenna diversity using maximal ratio combining (MRC), but with a 3dB power penalty (this power penalty comes from a fact that the transmit power per antenna is halved so that the total transmit power is kept the same).
Symbol decision is carried out as

\[
\begin{align*}
\tilde{d}_{e,n}(k) = \arg \min_{d_{e,n}} & \left[ -d_{e,n} \sum_{m=0}^{N-1} \left[ H(m)^{(e)}_{0,n}(k) \right]^2 + \left[ H(m)^{(e)}_{1,n}(k) \right]^2 \right]^{1/2} \, , \\
\tilde{d}_{o,n}(k) = \arg \min_{d_{o,n}} & \left[ -d_{o,n} \sum_{m=0}^{N-1} \left[ H(m)^{(o)}_{0,n}(k) \right]^2 + \left[ H(m)^{(o)}_{1,n}(k) \right]^2 \right]^{1/2} .
\end{align*}
\]

Then, after parallel-to-serial (P/S) conversion, data demodulation is performed to recover the transmitted data.

C. PA-DFCE

Figure 2 shows the PA-DFCE structure. Simultaneous channel estimation for the two transmit channels is described below. Firstly, we estimate the instantaneous channel gains of the two transmit channels by applying the reverse modulation and then the frequency-domain and time-domain averaging filter is applied to improve the channel estimation accuracy.

From Eq. (3), the channel gains, \( \sqrt{\mathbb{S} H^{(m)}_{0,n-1}(k)} \) and \( \sqrt{\mathbb{S} H^{(m)}_{1,n-1}(k)} \), can be expressed as

\[
\begin{align*}
\sqrt{\mathbb{S} H^{(m)}_{0,n-1}(k)} &= \frac{1}{2} \left[ R^{(m)}_{e,n-1}(k) d^{*}_{e,n-1}(k) - R^{(m)}_{o,n-1}(k) d^{*}_{o,n-1}(k) \right] \\
&\quad + \frac{1}{2} \left[ \Pi^{(m)}_{0,n-1}(k) d^{*}_{o,n-1}(k) - \Pi^{(m)}_{1,n-1}(k) d^{*}_{o,n-1}(k) \right] , \\
\sqrt{\mathbb{S} H^{(m)}_{1,n-1}(k)} &= \frac{1}{2} \left[ R^{(m)}_{e,n-1}(k) d^{*}_{o,n-1}(k) + R^{(m)}_{o,n-1}(k) d^{*}_{e,n-1}(k) \right] \\
&\quad - \frac{1}{2} \left[ \Pi^{(m)}_{0,n-1}(k) d^{*}_{o,n-1}(k) + \Pi^{(m)}_{1,n-1}(k) d^{*}_{e,n-1}(k) \right] ,
\end{align*}
\]

(8)

The above suggests that if the past data symbols, \( \tilde{d}_{e,n-1}(k) \) and \( \tilde{d}_{o,n-1}(k) \), are available, \( \sqrt{\mathbb{S} H^{(m)}_{0,n-1}(k)} \) and \( \sqrt{\mathbb{S} H^{(m)}_{1,n-1}(k)} \) can be simultaneously estimated by

\[
\begin{align*}
\tilde{H}^{(m)}_{0,n-1}(k) &= \frac{1}{2} \left[ R^{(m)}_{e,n-1}(k) \tilde{d}^{*}_{e,n-1}(k) - R^{(m)}_{o,n-1}(k) \tilde{d}^{*}_{o,n-1}(k) \right] \\
\tilde{H}^{(m)}_{1,n-1}(k) &= \frac{1}{2} \left[ R^{(m)}_{e,n-1}(k) \tilde{d}^{*}_{o,n-1}(k) + R^{(m)}_{o,n-1}(k) \tilde{d}^{*}_{e,n-1}(k) \right] ,
\end{align*}
\]

(9)

For frequency-domain filtering, the following simple averaging filter is applied:

\[
\tilde{H}^{(m)}_{0(n),0,n-1}(k) = \frac{\sum_{k=\alpha}^{\beta} \tilde{H}^{(m)}_{0(n),0,n-1}(k+k')}{2\alpha+1} ,
\]

where \( \alpha \) is the number of subcarriers on one side for frequency-domain averaging. Then, a time-domain first-order filtering using the forgetting coefficient \( \beta \) (0 ≤ \( \beta \) ≤ 1) is applied to obtain

\[
\hat{H}^{(m)}_{0(n),0,n-1}(k) = (1-\beta)\tilde{H}^{(m)}_{0(n),0,n-1}(k) + \beta \hat{H}^{(m)}_{0(n),0,n-1}(k) ,
\]

with \( \hat{H}^{(m)}_{0(n),0,n-1}(k) = 0 \), where \( \alpha \) and \( \beta \) depend on the frequency-selectivity and time-selectivity of the channel, respectively. In this paper, they are optimized by computer simulation.

Known pilot can be used as \( \tilde{d}_{e,n-1}(k) \) and \( \tilde{d}_{o,n-1}(k) \) in Eq. (9). Channel estimation is carried out using the STTD encoded pilot block. The frame structure is illustrated in Fig. 3. STTD encoded pilot block is transmitted at the beginning of the frame and followed by \( N \) data blocks. If \( N \) is large, the tracking ability of the channel estimation against time-varying fading tends to be lost. However, the use of small \( N \) increases the power loss due to pilot insertion. Therefore, we combine pilot assisted CE and DFCE to obtain the PA-DFCE. At the beginning of each frame, channel estimation for STTD decoding of the 1st block is carried out using STTD encoded pilot block. For channel estimation to be used in STTD decoding of the 2nd block onwards, symbol decisions of the previous STTD encoded block are fed back as pilot symbols for estimating the instantaneous channel gains. Using the above described PA-DFCE, more accurate channel estimation is possible while increasing the tracking ability against fast fading, compared to the use of STTD encoded pilot block only.

\[
\begin{align*}
\{ \mathbb{R}^{(m)}_{e(n),0,n-1}(k) \} \rightarrow \text{Reverse mod.} \rightarrow \text{Averaging filter} \rightarrow \hat{H}^{(m)}_{0(n),0,n-1}(k) \\
\{ \mathbb{R}^{(m)}_{e(n),1,n-1}(k) \} \rightarrow \text{Averaging filter} \rightarrow \hat{H}^{(m)}_{1(n),1,n-1}(k) \\
\{ \mathbb{R}^{(m)}_{o(n),0,n-1}(k) \} \rightarrow \text{Averaging filter} \rightarrow \hat{H}^{(m)}_{0(n),0,n-1}(k) \\
\{ \mathbb{R}^{(m)}_{o(n),1,n-1}(k) \} \rightarrow \text{Averaging filter} \rightarrow \hat{H}^{(m)}_{1(n),1,n-1}(k) \\
\end{align*}
\]

Fig. 2 Structure of PA-DFCE.

\[
\begin{align*}
\text{Frame} & \quad \text{Block 0 } \quad \text{Block 1 } \quad \text{Block 2 } \\
\text{STTD encoded pilot block} & \quad \text{STTD encoded data blocks} \\
\text{Antenna 0 } \{ \tilde{d}_{e(n),0,n-1}(k) \} & \quad \{ \tilde{d}_{o(n),0,n-1}(k) \} \quad \{ \tilde{d}_{e(n),1,n-1}(k) \} \\
\text{STTD encoded pilot block} & \quad \text{STTD encoded data blocks} \\
\text{Antenna 1 } \{ \tilde{d}_{e(n),0,n-1}(k) \} & \quad \{ \tilde{d}_{o(n),0,n-1}(k) \} \quad \{ \tilde{d}_{e(n),1,n-1}(k) \} \\
\end{align*}
\]

Fig. 3 Frame structure of OFDM with STTD.

III. COMPUTER SIMULATION

Table 1 shows the computer simulation conditions. We assume \( K=256 \) subcarriers, GI of \( N_g=32 \) samples, and quadrature-phase shift keying (QPSK) data modulation. A \( T_s \)-spaced \( L=16 \)-path frequency-selective Rayleigh fading channel having a uniform power delay profile is assumed. The normalized maximum Doppler frequency \( f_{DT} = 0.0003 \sim 0.007 \) is assumed, where \( T \) is the OFDM signaling interval including GI (i.e., \( T = (K + N_g)T_s \)).

<table>
<thead>
<tr>
<th>Table 1 Simulation conditions</th>
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<tbody>
<tr>
<td>OFDM</td>
</tr>
<tr>
<td>Data modulation</td>
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<tr>
<td>Number of FFT points</td>
</tr>
<tr>
<td>GI</td>
</tr>
<tr>
<td>Fading channel</td>
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<tr>
<td>Number of paths</td>
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<tr>
<td>Power delay profile</td>
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<tr>
<td>Normalized Doppler frequency</td>
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<tr>
<td>Number of receive antennas</td>
</tr>
</tbody>
</table>
A. Optimum values of $\alpha$ and $\beta$

Figure 4 shows the effects of $\alpha$ and $\beta$ with $f_D T$ as a parameter. It can be seen from Fig. 4(a) that the optimum value of $\alpha$ is $\alpha=1$ (i.e., the number of averaging subcarriers are 3) irrespective of $f_D T$. In Fig. 4(b), the average BER with $\alpha=1$ is plotted as a function of $\beta$. The optimum $\beta$ is about 0.8 when $f_D T=0.0003$ and 0.5 when $f_D T=0.001$, respectively. The faster the fading becomes, the smaller the optimum $\beta$ becomes to better track the fading. With $f_D T=0.003$ and 0.007, the optimum $\beta$ is 0.2 and 0 (no time averaging). In the following simulation, $\alpha=1$ is always used and $\beta=0.8$, 0.5, 0.2 and 0 is used when $f_D T=0.0003$, 0.001, 0.003 and 0.007, respectively.

![Graph showing effect of $\alpha$ and $\beta$.](image)

B. Average BER performance

Figure 5 shows the average BER performance as a function of the average received $E_b/N_0$ with the number $N$ of blocks in a frame as a parameter when $f_D T=0.001$. For comparison, the BER performance without DF (only the pilot block is used) and that with differential STTD that requires no channel estimation are also plotted. It can be clearly seen that the average BER performance can be significantly improved by PA-DFCE compared to the case without DF. Without DF, the error floor exists because of the inability to track fast fading; however, the PA-DFCE produces no error floor. Also seen is that the PA-DFCE provides better BER performance than differential STTD. When $N=64$, an $E_b/N_0$ degradation with DFCE from ideal channel estimation is about 2dB for achieving a BER=$10^{-3}$ and is smaller by about 1dB than differential STTD. Using receive antenna diversity ($N_r=2$), DFCE can achieve an average BER performance close to the ideal channel estimation by 1dB and can achieve better performance by about 2dB compared with the differential STTD. Even with $N=128$, almost no performance degradation is seen.
Figure 6 shows the average BER performance with $f_D T$ as a parameter when $N=16$. As fading becomes faster, the achievable BER performance with PA-DFCE degrades since the tracking ability of the PA-DFCE tends to be lost. However, without antenna diversity, if $f_D T < 0.003$, the STTD with PA-DFCE provides better or at least the same BER performance compared to differential STTD. When two-antenna diversity reception ($N_r=2$) is used, the PA-DFCE provides superior BER performance to differential STTD even in a fast fading environment of $f_D T = 0.007$, which is equivalent to a moving speed of 295km/h for a carrier frequency of 5GHz and a transmitted data rate of 100Mbps.

![Figure 6. Effect of $f_D T$.](image)

**IV. CONCLUSION**

In this paper, a pilot-assisted decision feedback channel estimation (PA-DFCE) suitable for STTD in OFDM signal transmission was proposed. The PA-DFCE can simultaneously estimate the transmit channels by transmitting STTD encoded pilot. The instantaneous channel gains are estimated by reverse modulation using STTD encoded pilot block or decision feedback of the previous decision results. Noise reduction in the channel estimation is achieved by averaging noisy instantaneous channel estimates in time- and frequency-domain. Using the decision feedback, very good tracking ability is achieved. It has been confirmed by the computer simulation that even in a fast fading environment, STTD using the PA-DFCE provides superior BER performance to differential STTD that requires no channel estimation.

**REFERENCES**


