Low-Complexity Beam Allocation for Switched-Beam Based Multiuser Massive MIMO Systems

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Best Wishes to Professor Fumiyuki Adachi, Father of Wideband CDMA [1].

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Motivation

- What do people usually do when they are travelling, waiting for a bus or the food, ....?

- Rapid growth of smartphone users
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- What are the popular smartphone applications (Apps)?
  - Online gaming
  - High-definition video streaming
  - Social networking
  - Rapid development of high-data-rate applications
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Demand for high data rates
What’s the maximum data rate we can get?

Shannon Capacity

Maximum achievable data rate:

\[ C = B \log_2 \left( 1 + \frac{S}{N} \right) \text{ (bit/s)} \]

- \( B \): Communication bandwidth;
- \( S \): Received signal power;
- \( N \): Noise power.

- Shannon capacity \( C \) \( \uparrow \) \( \Rightarrow \)
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- Shannon capacity \( C \uparrow \) ⇒ Communication bandwidth \( B \uparrow \)
- Received signal power \( S \uparrow \)
Bandwidth $B \uparrow$: 

5G MOBILE COMMUNICATION SYSTEMS
**5G Mobile Communication Systems**

- **Bandwidth** $B \uparrow$:

  \[
  \lambda = 1\text{m} \quad \lambda = 10\text{cm} \quad \lambda = 1\text{cm} \quad \lambda = 1\text{mm}
  \]
5G Mobile Communication Systems

- Bandwidth $B \uparrow$:

  $\lambda = 1m \quad \lambda = 10cm \quad \lambda = 1cm \quad \lambda = 1mm$

  millimeter wave (mmWave) communication
5G Mobile Communication Systems

- Received signal power $S \uparrow$:
  - Distributed antenna systems (minimum access distance $S \downarrow$)[2]–[4]
  - Deploy large antenna array at the base-station (BS)

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**Massive multiple-input-multiple-output (MIMO)**


Massive MIMO Beamforming

- Enhance the received signal power ⇒ Enlarge the coverage of each BS
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Increase the number of antenna elements by $m$ times
- Beam gain increased by $m$;
- Beam width decreased by $1/m$.

$\lambda$: propagation wavelength

Beamforming technologies: digital beamforming & analog beamforming
**Beamforming Technology**

- **Increase** the number of antenna elements by $m$ times
  - Beam **gain** increased by $m$;
  - Beam **width** decreased by $1/m$.

\[ \lambda/2 \quad \text{W} \quad G \quad \lambda/2 \quad \text{W/2} \quad 2G \]

$m = 2$  \[ \Rightarrow \]

- Beamforming technologies: **digital** beamforming & **analog** beamforming

$\lambda$: propagation wavelength

NTT DOCOMO
Digital Beamforming vs. Analog Beamforming

- Digital beamforming: number of RF chains & digital-to-analog converters (DACs) = number of antennas \( N \)

- Analog Beamforming: number of RF chains & DACs = number of users \( K \)

For a massive MIMO system with \( N \gg K \), analog beamforming is of low cost and with low power consumption.
**Analog Beamforming: Switched-Beam Scheme**

- Switched-Beam Scheme: Beam pattern is fixed. (eg. Butler method,...)

How to allocate beams to users to maximize sum data rate?
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Analog Beamforming: Switched-Beam Scheme

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How to allocate beams to users to maximize sum data rate?
**RELATED WORK**

Random beamforming based systems (e.g. [5]):

- **Assumptions:**
  - The number of users $K$ is assumed to be much larger than the number of beams $N$ to exploit multiuser diversity.
  - All the beams are used for data transmission with equal power allocation.

- **Beam allocation scheme:**
  1. Each user measures the received signal-to-interference-plus-noise ratios (SINRs) on the $N$ beams and then feeds back the maximum SINR and the corresponding beam index to the BS;
  2. After receiving feedback from all users on all beams, the BS assigns each beam to the best user with the highest SINR to maximize the sum data rate.

Adopted in Massive MIMO Systems?

The beam allocation scheme in [5] cannot be directly used in switched-beam based massive MIMO systems

Massive MIMO system: $N \gg K$

Some of the beams may not be used for data transmission.

The beams used for data transmission vary when channel condition changes.

Impossible for each user to obtain the received SINR on each beam without being informed the beam allocation result.

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How to allocate beams in a massive MIMO system with $N \gg K$?
A low-complexity beam allocation (LBA) algorithm is proposed to maximize the sum data rate for a switched-beam based massive MIMO system ($N \gg K$).

Our proposed LBA algorithm achieves nearly optimal sum data rate with a linear complexity $O(KN)$.

Average service ratio, i.e., the average percentage of users that can be served simultaneously is theoretically derived as a monotonic increasing function of the ratio $N/K$.

$N$: number of beams; $K$: number of users.
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System Model

- K users are uniformly distributed within a circular cell and a linear array with \( N \) equally spaced antenna elements is employed at the central base-station (BS).
- Butler method is used to generate fixed beams.
- Light-of-sight (LoS) channel at mmWave frequencies is assumed.
**Problem Formulation**

- The total transmission power is fixed and equally allocated to the beams selected for data transmission.

### Sum Data Rate Maximization

$$\max_{\{c_{k,n}\}} \quad R_s = \sum_{k=1}^{K} R_k$$  

Maximize sum data rate

s.t.  

$$\sum_{n=1}^{N} c_{k,n} \leq 1, \forall k$$  

Each user can only use one beam

$$\sum_{k=1}^{K} c_{k,n} \leq 1, \forall n$$  

Each beam can only serve one user

$$c_{k,n} \in \{0, 1\}, \forall k, \forall n$$

$R_k$: Achievable data rate of user $k$

$c_{k,n}$: Indicator for beam allocation.
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Optimal Beam Allocation

- Optimal beam allocation can be obtained via brute-force (exhaustive) search.

- Complexity: $O(N^K)$

- For a massive MIMO system with a very large $N$, the complexity is prohibitively high.

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**Our Goal**

Develop a beam allocation algorithm with low complexity
Optimal Beam Allocation

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Develop a beam allocation algorithm with low complexity
Low-Complexity Beam Allocation (LBA)

For a multiuser massive MIMO system with $N \gg K > 1$:

- Beams are very narrow & overlap of two beams is small.
- Only some of the beams are used for data transmission.

Ignore the effect of inter-beam interference

Decouple the beam allocation problem into two parts:

- Beam-user association;
- Beam allocation.
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1. Beam-user association;
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**LBA Algorithm**

- **Two-step LBA algorithm**
  1. **Beam-user association**: Each user is associated with its best beam with the largest beam gain.
  2. **Beam allocation**: Each beam is allocated to its best associated user with the highest received signal-to-noise ratio (SNR).

- **Complexity**: $O(KN)$

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**LBA Algorithm**

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Sum Data Rate

- Sum data rate $R_s$

![Graph](image)

- Optimal Brute-Force Search
- LBA

(a) $K = 6$.  
(b) $K = 10$.  

$N = 16$. $P_t/\sigma^2 = 20\text{dB}$. $N$: number of beams; $K$: number of users

- Our proposed algorithm achieves nearly optimal sum data rate.
- Sum data rate is sensitive to the users’ positions.
Sum Data Rate

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**Average Sum Data Rate**

- Average sum data rate over users’ positions $\bar{R}_s$

![Graph](image)

(a) $N = 64$. $P_t/\sigma_2^2 = 20\text{dB}$.  

(b) $K = 4$. $P_t/\sigma_2^2 = 20\text{dB}$.

$N$: number of beams; $K$: number of users

- $K \uparrow \Rightarrow \bar{R}_s \uparrow$; $N \uparrow \Rightarrow \bar{R}_s \uparrow$
- Rate gap $\uparrow$ as $N \downarrow$.  
  $N \downarrow \Rightarrow$ beam width $\uparrow \Rightarrow$ inter-beam interference $\uparrow \Rightarrow$ rate gap $\uparrow$
Average Sum Data Rate

- Average sum data rate over users’ positions $\bar{R}_s$

- $\bar{R}_s$ (bit/s/Hz)

- Optimal Brute-Force Search

- $LBA$

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Service Ratio

- Not all the users can be always served simultaneously.

**Step 1**: Each user is associated with the beam with the largest directivity.

**Step 2**: Each associated beam is allocated to the user with the highest received signal power.

- Service ratio $P_s$:
  \[ P_s = \frac{\text{No. of Served Users}}{K} \]

- Average service ratio over users’ positions $\bar{P}_s$:
  \[ \bar{P}_s \approx f \left( \frac{N}{K} \right) \]
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Unserved

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$$\bar{P}_s \approx f \left( \frac{N}{K} \right)$$
Average Service Ratio

- Average service ratio over users’ positions $\bar{P}_s$

\[ N = 512. \quad P_t/\sigma^2 = 20\text{dB}. \]

$N$: number of beams;

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- The analysis serves as a good approximation of $\bar{P}_s$.

- $\bar{P}_s$ increases with the ratio $N/K$.
  - With $N/K \ll 1$, $\bar{P}_s \approx N/K$; as $N/K \to \infty$, $\bar{P}_s \to 1$. 
Average Service Ratio

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**Summary**

- **Beam allocation** in switched-beam based mmWave massive MIMO systems is studied.
  
  - Propose a low-complexity beam allocation (LBA) algorithm.
  
  - Nearly optimal performance can be achieved by adopting the proposed LBA algorithm with very low complexity $O(KN)$.
  
  - Investigate the average service ratio with our proposed algorithm, which is a monotonic increasing function of the ratio $N/K$. 
Thank you!