Compressed Sensing for Multiple Access

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A typical wireless sensor network contains a large number of sensor nodes.

However, spectrum resources and system power are limited!
Sparsity

Most signals in our natural world are sparse

Obtained $M < N$ measurements


Use $l_1$ norm reconstruction to recover sparsest coefficients satisfying

$$\hat{s} = \arg \min \| s \|_1 \quad \text{such that} \quad y = \Theta s$$
Compressed sensing has attracted significant interests in science and engineering. [http://dsp.rice.edu/cs](http://dsp.rice.edu/cs)

The communications research community has applied CS to many problems such as sparse channel estimation, cognitive radio, channel coding, etc.

CS application to wireless sensor networks was pioneered by Bajwa, Novak, *et al.* in *IPSN* 2006.
Existing Works (Contd)

- Random Access Compressed Sensing (RACS)
- CS-based Multi-User Detection
- CS-based Multiple Access
For a WSN, let the fusion center (FC) simply **discards** the erroneous packets as long as [F. Fazel et al., *JSAC* 2011]

i) the selected subset is chosen uniformly at random

ii) there are sufficiently many useful packets remaining to allow for the reconstruction of the field
Average number of collision-free packets versus probability of active $p$ for each sensor node when $N = 1000$.

Then utilize the sparsity in frequency domain to recover all the data.
CS-based multi-user detection exploits the fact that, in some wireless systems, the number of active users may be small relative to the total number of users in the system [Y. Xie et al., Trans. IT 2013]
For the wireless system with random data traffic, transmitter identities can be compressed into data transmission based on the assumption that the transmitted signal vector from all the users is sparse. [R. Mao et al., J. of Commun., 2010]

Why CS?

The key point here is that it is not necessary to transmit the location information of the zero or non-zero elements in a signal vector. These can be obtained after CS reconstruction.
CS Multiple Access for WSN

Main Parts

1. Traffic Model
2. Signal Model
3. CS-based Symbol Reconstruction
4. Sensing Matrix Selection
5. The impact of SNR on CS
6. CS for Network Data Recovery
Sparsity in WSN

Where the sparsity comes from?

**Spatial correlation** due to the closeness of sensors’ geographical locations

**Temporal correlation** due to the smooth variations of the real world signal

It is not necessary to let all the sensor nodes be active. **Power saved!**

Small number of active sensors makes the signal vector **sparse!**
Traffic Model
Signal Model

The received signal at the $n$-th time slot is given by

$$y(n) = (\hat{\Phi}'(n) \circ H(n))x + w(n)$$

where

$$\hat{\Phi}'(n) = \left[\begin{array}{c} \Phi(n)^T \\ \vdots \\ \Phi(n)^T \end{array}\right]^T \in \mathbb{C}^{(M_c \times M_r) \times N}$$

Keep retransmitting the same packet till the end of the $p$-th time slot, the received signal can be written as

$$z = \Psi x + W$$

where

$$z = [y^T(1) \ldots y^T(p)]^T$$

$$\Psi = \left[ (\hat{\Phi}'(1) \circ H(1))^T \ldots (\hat{\Phi}'(p) \circ H(p))^T \right]^T$$

$$W = [w^T(1) \ldots w^T(p)]^T$$
Symbol Reconstruction

Solve $l_1$ norm minimization problem $P1$ below

$$\minimize_x \|x\|_1 \quad \text{subject to} \quad \|z - \Psi x\|_2 \leq \varepsilon$$
Sensing Matrix Selection

Case 1: Single receive antenna ($M_r=1$), channel static across frequency and time

$$z = [\Phi'(1)^T \cdots \Phi'(p)^T]^T x + W$$

where $x$ involves data and channel information.

Case 2: Channel static in one frame, but independent over frequency and antennas

$$z = [(\Phi'(1) \circ H)^T \cdots (\Phi'(p) \circ H)^T]^T x + W$$

where $H$ is a random channel matrix with independent complex Gaussian random variables as elements.

Case 3: Channel independent across time, frequency and spatial antennas

$$z = [H^T(1) \cdots H^T(p)]^T x + W$$

where the channel matrix becomes the actual sensing matrix for CS.
The capacity of the proposed method in one time frame can be treated as the maximum allowable number of active sensors $r$ in this time frame, multiplied by the effective data rate $L/T_f$, i.e., the capacity

$$C = Lr / T_f$$

where $L$ represents the length of the data packet, $T_f$ is the duration of one time frame. The capacity can also be expressed as

$$C = O(RM_cM_r / \log N)$$

where $M_c$ represents the number of sub-channels, $M_r$ is the number of antennas at the receiver, and $R$ is the transmission rate of one channel.
Reconstructed symbols in constellation diagram for QPSK (SNR = 0 dB).
The Impact of SNR

Reconstructed symbols in constellation diagram for QPSK (SNR = 12 dB).
Reconstructed symbols in constellation diagram for QPSK (SNR = 24 dB).
Network Data Recovery

Utilizing sparsity from spatial correlation

\[ d_A(l) = U(l)d(l) = U(l)F^{-1}b(l) \]

Solve P2

\[
\begin{align*}
\text{minimize} & \quad \|b(l)\|_1 \\
\text{subject to} & \quad d_A(l) = U(l)F^{-1}b(l)
\end{align*}
\]

Where \( F^{-1} \) represents the inverse DFT matrix and \( b(l) \) is sparse. \( U(l) \) contains several randomly selected rows of identity matrix representing the address information of the active nodes.
Network Data Recovery

Utilizing sparsity from temporal correlation

\[ t_i = U_i d_i = U_i W^{-1} v_i \]

Solve P3

\[
\begin{align*}
\text{minimize} & \quad \|v_i\|_1 \\
\text{subject to} & \quad t_i = U_i W^{-1} v_i
\end{align*}
\]

Where \( W^{-1} \) represents the orthogonal inverse DWT matrix and \( v_i \) is sparse. \( U_i \) contains several randomly selected rows of identity matrix representing the index of the time frame during which the sensor readings of sensor node \( i \) are successfully recovered previously.
The data recovery process utilizing spatial and temporal correlations.

\[
\begin{array}{cccccccc}
  d_1(1) & d_1(2) & d_1(3) & d_1(4) & d_1(5) & \cdots & d_1(l) \\
  d_2(1) & d_2(2) & d_2(3) & d_2(4) & d_2(5) & \cdots & d_2(l) \\
  d_3(1) & d_3(2) & d_3(3) & d_3(4) & d_3(5) & \cdots & d_3(l) \\
  d_4(1) & d_4(2) & d_4(3) & d_4(4) & d_4(5) & \cdots & d_4(l) \\
  d_5(1) & d_5(2) & d_5(3) & d_5(4) & d_5(5) & \cdots & d_5(l) \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  d_N(1) & d_N(2) & d_N(3) & d_N(4) & d_N(5) & \cdots & d_N(l) \\
\end{array}
\]

- **Recovered packets through CS-based symbol recovery** (P1)
- **Recovered packets utilizing spatial correlation** (P2)
- **Recovered packets utilizing temporal correlation** (P3)
- **Data unavailable**
Simulation Examples
Simulation Examples

![Graph showing the required channel capacity (kbps) vs. sparsity from spatial correlation for different values of N: N=100, N=200, and N=500.](chart.png)
Simulation Examples
Simulation Examples

Reconstructed symbols in the constellation diagram of QPSK with SNR = 24 dB.

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Simulation Examples
Simulation Examples

Average number of successfully received data packets vs. Channel capacity (kbps)

- r=10 CS
- r=10 CSMA
- r=20 CS
- r=20 CSMA
Conclusion

- The spatial and temporal correlation inherent in the WSN data can be leveraged to reduce the total power consumption of the network.

- Three levels of CS have been applied to solve multiple access and network data recovery problems.

- CS-MAC outperforms CSMA in throughput under certain conditions depending on the number of total nodes, active nodes, resources available, at the price of higher computational complexity.
Thank you for your attention