Low-complexity Algorithms for MIMO Multiplexing Systems

彭 薇

Outline

- Introduction
- QRD-M algorithm
- Algorithm I: to reduce the number of surviving paths.
- Algorithm II: to reduce the number of candidates for each transmitted signal.
- Algorithm III: to reduce both the number of surviving paths and the number of candidates.
- Summary

Outline

Introduction

MIMO structure

Introduction

MIMO structure

Detection algorithms
- Linear detection algorithms: zero-forcing (ZF) estimation and minimum mean square error (MMSE) estimation.
- Non-linear detection algorithms: maximum likelihood (ML) detection, sphere decoding algorithm (SD), successive interference cancellation (SIC) detection and QR decomposition associated M (QRD-M) algorithm.

\[ Y = HX + N \]

- \( Y \): The received signal vector.
- \( H \): The channel matrix.
- \( X \): The transmitted signal vector.
- \( N \): The additive white Gaussian noise.
**QRD-M algorithm**

Assuming \[ y = [x(k,0+\cdot\cdot\cdot), x(k,0)] + n \]

QR decomposition and left multiply the received signal vector with \( Q^H \).

The QR decomposition based ML detection is achieved by

\[ \hat{x} = \arg \min_{\{c_1, \ldots, c_N\}} D(c_1, \ldots, c_N) \]

where \( D(c_1, \ldots, c_N) \) is the accumulated squared Euclidean distance (ASED) of the path \( \{c_1, \ldots, c_N\} \).

\[ D(c_1, \ldots, c_N) = d(c_1) + d(c_1, c_2) + \cdots + d(c_1, \ldots, c_N) \]

Two approaches to reduce the complexity of QRD-M algorithm

- To reduce the number of surviving paths in each stage [Kim, Yue and Iltis, 2005; Kawai, Higuchi, Maeda and Sawahashi, 2006].
- To reduce the number of candidates of each transmitted signal [Higuchi, Kawai, Maeda and Sawahashi, 2004; Nagayama and Hattori, 2006; Im, Kim and Cho, 2007].

The computation involved in the QR decomposition will be identical for all the QRD-M algorithms. Therefore, the differences in the computational complexity between different QRD-M algorithms are determined by the number of SED calculations, which are also the dominant computational task. The computational complexity of the proposed and the existing QRD-M algorithms are therefore approximately evaluated in terms of the number of SED calculations.
**Algorithm I:** to reduce the number of surviving paths at each stage

- **Idea:** to determine the number of surviving paths in each stage adaptively according to the channel condition of the transmitted signal.

- **Step 1:** Partition the probability density function of the diagonal element $|r_{ii}|$

$$\int_{0}^{r_{ii}^{\text{th}-1}} p(|r_{ii}|) d(|r_{ii}|) = 1/K_{n<in},$$

$$\int_{r_{ii}^{\text{th}}}^{r_{ii}^{\text{th}+1}} p(|r_{ii}|) d(|r_{ii}|) = 1/K_{n=1},$$

... 

$$\int_{r_{ii}^{\text{th}k-1}}^{r_{ii}^{\text{th}k}} p(|r_{ii}|) d(|r_{ii}|) = 1/K_{n=r^{*}},$$

where $r_{ii}^{\text{th}k}$ is the threshold of the $k$-th sub-region.

**Algorithm I: to reduce the number of surviving paths**

1. The p.d.f of $|r_{ii}|$ is equally partitioned.
2. The instantaneous $|r_{ii}|$ is mapped to the sub-regions.
3. The number of surviving paths is determined according to the index of the sub-region.

**Simulation conditions of algorithm I**

<table>
<thead>
<tr>
<th>System</th>
<th>Modulation</th>
<th>Number of transmit and receive antennas</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel</td>
<td>16QAM or 64QAM</td>
<td>3 transmit and 3 receive antennas</td>
</tr>
<tr>
<td></td>
<td>Channel gain</td>
<td>i.i.d Rayleigh distributed.</td>
</tr>
<tr>
<td></td>
<td>Noise</td>
<td>Complex Gaussian, zero mean, unit variance.</td>
</tr>
<tr>
<td></td>
<td>CSI</td>
<td>Least squares channel estimation</td>
</tr>
<tr>
<td>Reference algorithm</td>
<td>QRD-M algorithm with fixed number of surviving paths</td>
<td>[Kim and Ilitis, 2002]</td>
</tr>
</tbody>
</table>
Algorithm I: to reduce the number of surviving paths

Simulation results of algorithm I

![Graph showing BER vs SNR for various detection methods with different modulation schemes and QRD-M detection configurations.]

Algorithm II: to reduce the number of candidates for each transmitted signal

- Idea: to select part of the constellation points instead of all the constellation points to save the computation.
- Reference algorithm [Nagayama and Hattori, 2006]
  - Step 1: calculate the signal estimate on each path.
  - Step 2: determine which quadrant does the signal estimate falls into.
  - Step 3: choose the candidates according to the quadrant.
The proposed algorithm:

- Step 1: calculate the signal estimate on each path.
- Step 2: determine a circle centering at the signal estimate.
- Step 3: choose the constellation points within the circle.

Simulation conditions of algorithm II:

<table>
<thead>
<tr>
<th>System</th>
<th>Modulation</th>
<th>16QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of transmit and receive antennas</td>
<td>4 transmit and 4 receive antennas</td>
<td></td>
</tr>
<tr>
<td>Channel gain</td>
<td>i.i.d Rayleigh distributed.</td>
<td></td>
</tr>
<tr>
<td>Noise</td>
<td>Complex Gaussian, zero mean, unit variance.</td>
<td></td>
</tr>
<tr>
<td>CSI</td>
<td>Least squares channel estimation</td>
<td></td>
</tr>
<tr>
<td>Reference algorithm</td>
<td>QRD-M algorithm to reduce the number of candidates by the quadrant of signal estimate in the constellation</td>
<td>[Nagayama and Hattori, 2006]</td>
</tr>
</tbody>
</table>

Simulation results of algorithm II:

- 16QAM modulated (4,4) system
- BER vs SNR (dB)
- Complexity of the reference algorithm
- Complexity of the proposed algorithm
Algorithm III: to reduce both the number of surviving paths and the number of candidates for each transmitted signal

- This algorithm has combined both approaches to reduce the complexity.
- Reduction of the number of candidates for each transmitted signal.

Let \( \text{path}(c_1, \ldots, c_i, \ldots, c_M) \) be the correct path at the \((i-1)^{\text{th}}\) stage.

\[
\hat{x}(\text{path}(c_1, \ldots, c_i, \ldots, c_M)) = \frac{1}{2^M} \sum_{j=1}^{2^M} \tau_j \left( x_j - \hat{x}(\text{path}(c_1, \ldots, c_{i-1}, c_i, \ldots, c_M)) \right)
\]

The point within the circle centering at \( \hat{x}(\text{path}(c_1, \ldots, c_i, \ldots, c_M)) \) with radius \( r = \alpha \sigma / \sqrt{N_0} \) has high probability to be the correct candidate.

Only the constellation points within the circle around the signal estimate will be selected as candidates. The constellation points outside the circle will be discarded.
### Implementation for algorithm II and III

- **How to determine the circle?**
  - To calculate the real time Euclidean distance between the signal estimate and the constellation points will cause additional calculations.

  This work is done off-line by using a look up table.

### Simulation conditions of algorithm III

<table>
<thead>
<tr>
<th>System</th>
<th>Modulation</th>
<th>16QAM and 64QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of antennas</td>
<td>3 transmit and 3 receive antennas, 4 transmit and 4 receive antennas</td>
<td></td>
</tr>
<tr>
<td>Channel</td>
<td>Channel gain</td>
<td>i.i.d Rayleigh distributed.</td>
</tr>
<tr>
<td></td>
<td>Noise</td>
<td>Complex Gaussian, zero mean, unit variance.</td>
</tr>
<tr>
<td></td>
<td>SNR</td>
<td>0dB-30dB</td>
</tr>
<tr>
<td></td>
<td>CSI</td>
<td>Least squares channel estimation</td>
</tr>
<tr>
<td>Reference algorithms</td>
<td>QRD-M algorithm with fixed number of surviving paths</td>
<td>[Kim and Itis, 2002]</td>
</tr>
<tr>
<td></td>
<td>QRD-M algorithm to reduce the number of surviving paths in each stage</td>
<td>[Kawai, Higuchi, Maeda and Sawahashi, 2006]</td>
</tr>
<tr>
<td>Parameters</td>
<td>16QAM modulated 4,4 system</td>
<td>( \lambda = 1.5, 2, 2.5; k=2, 3, 4; M=12 )</td>
</tr>
<tr>
<td></td>
<td>64QAM modulated 3,3 system</td>
<td>( \lambda = 2, 2.5, 3; k=2, 3, 4; M=12 )</td>
</tr>
</tbody>
</table>

### Simulation results of algorithm III: the effects of \( \lambda \)

The effect of \( \lambda \) on the BER performance for a 16QAM modulated system, \( N_t = 4, N_r = 4 \)
Simulation results of algorithm III: the effects of $\lambda$

The effect of $\lambda$ on the computational complexity for a 16QAM modulated system, $N_r = 4, N_f = 4$

Simulation results of algorithm III: comparison with the reference algorithms

The proposed algorithm can achieve more than an order of reduction in computational complexity at SNR=16dB.

The BER performance for a 64QAM modulated system, $N_r = 3, N_f = 3$
Simulation results of algorithm III: comparison with the reference algorithms

The proposed algorithm can achieve a 93% reduction in the computational complexity at SNR=20dB.

The computational complexity for a 64QAM modulated system, \( N_1 = 3, N_1 = 3 \)

Summary

Three algorithms are proposed to reduce the computational complexity of QRD-M algorithm. Algorithm I reduces the number of surviving paths at each stage; Algorithm II reduces the number of candidates for each transmitted signal; Algorithm III reduce both the number of surviving paths and the number of candidates for each transmitted signal.

The three algorithms can achieve a reduction of complexity of QRD-M algorithms when compared with the existing algorithms, and they have different applications:

- When the channel state information (CSI) and channel distribution information (CDI) are available at the receiver, Algorithm I can be applied.
- When only the CSI is available at the receiver, Algorithm II can be applied.
- When CSI and the noise power are available at the receiver, Algorithm III can be applied.

Thank you!