Multiuser MIMO Systems: Top-Down or Bottom-Up Design?

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Outline

• Background
• ZF MIMO receiver vs. Optimal MIMO receiver (single user case)
• ZF MIMO receiver vs. Optimal MIMO receiver (multiuser case)
• Conclusion

MIMO Techniques

• Independent parallel transmit and receive antenna pair
• MIMO antenna techniques
  – boost channel capacity
  – enhance link reliability
  – reject strong interference

Diversity-Multiplexing Tradeoff

• Diversity -> improve link reliability by replicas

• Multiplexing -> enhance data rate by multiplexing

subchannel interference canceling is required
Scheduling Techniques

- Through periodically selecting the best user to serve, the system performance is improved by exploiting multiuser diversity or cooperative diversity.
- Ordered Statistics is the fundamental mathematical techniques for analyzing the scheduling wireless systems.

A Multiuser Multiplexing-based MIMO Scheduling System

- Perfect SNR estimation and noiseless feedback
- $H = [h_{ij}(k)]$, each $h_{ij}(k)$ is subject to Nakagami fading
- The BS selects the target user with highest effective SNR
  $$k^* = \arg \max_k \gamma_k$$

Issue for a Spatial Multiplexing-based MIMO

- Diversity-multiplexing tradeoff in a point-to-point MIMO system
  - multiplexing gain comes at the price of diversity gain [Zheng & Tse ‘03]
  - may translate into smaller coverage areas if the SM MIMO scheme is used [Catreux & Greenstein ‘03]
- The coverage is defined as the maximum distance at which the link suffices for maintaining a required receive SNR $\gamma_0$, with a probability, say 90% at least

The SWNSF Scheduling Game

SWNSF: strongest-weakest-normalized-subchannel-first

$\lambda_{1,1}$, $\lambda_{1,2}$, $\lambda_{1,3}$, $\lambda_{2,1}$, $\lambda_{2,2}$, $\lambda_{2,3}$, $\lambda_{3,1}$, $\lambda_{3,2}$, $\lambda_{3,3}$
**Effect of SWNSF Scheduling on \( \lambda_{\min} \)**

SWNSF scheduling enhances the output SNR of the weakest subchannel

\[ F_{\lambda_1}(\lambda) = [F_{\lambda_i}(\lambda)]^K \]

**Effect of SWNSF Scheduling on Other \( \lambda_i \)**

**Effect of SWNSF Scheduling on Coverage**

**Coverage Extension & Capacity Improvement**

<table>
<thead>
<tr>
<th>System</th>
<th>( K = 20 )</th>
<th>( K = 50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 1 ) SISO</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( N = 2 ) MIMO with RR scheduling</td>
<td>0.71</td>
<td>0.71</td>
</tr>
<tr>
<td>( N = 2 ) MIMO with SWNSF scheduling</td>
<td>1.52</td>
<td>1.65</td>
</tr>
<tr>
<td>( N = 3 ) MIMO with RR scheduling</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>( N = 3 ) MIMO with SWNSF scheduling</td>
<td>1.24</td>
<td>1.55</td>
</tr>
</tbody>
</table>
Main Observations

• Scheduling can effectively improve the coverage and capacity of multiplexing-based MIMO systems.
• “Soft coverage” concept allows to improve the coverage of the multiplexing-based MIMO system without increasing transmit power.

Motivation

• There are various kinds of MIMO receivers.
• Also, there are many higher-layer (MAC) scheduling schemes.
• Should we optimize MIMO receiver first, or MAC-layer scheduling receiver first?
• Can scheduling influence the physical layer design of MIMO receiver?

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Zero-Forcing MIMO receiver

- The ZF receiver is known to suffer from the effect of noise enhancement.
- ZF MIMO receiver is not an attractive solution from the perspective of performance.
- However, ZF MIMO receiver is simple and easy to be implemented.
- Question: What is the maximum achievable throughput for the ZF-MIMO receiver?

Output SNR for the ZF receiver

- The output SNR at the n-th subchannel after the zero-forcing processing is:
  \[ \gamma_n = \frac{\rho}{N_t \left(\mathbf{H}^H \mathbf{H}\right)^{-1} nn} \]
- \( \gamma_n \) is a Chi-squared distributed random variable with d.o.f. \( 2 (N_r - N_t + 1) \)

Achievable throughput for the ZF-MIMO receiver

\[
C_{zf} = \sum_{n=1}^{N_t} E[\log (1 + \gamma_n)] \\
= N_t \int_0^\infty \log (1 + \gamma_n) f_{\gamma_n}(\gamma_n) d\gamma_n \\
= N_t e^{N_r/\rho} \frac{N_r - N_t + 1}{\rho^{N_r - N_t + 1}} \sum_{n=1}^{N_r - N_t + 1} \left(\frac{\rho}{N_t}\right)^n \Gamma \left( n - N_r + N_t - 1, \frac{N_t}{\rho} \right)
\]
Optimal MIMO receiver

- Achievable throughput for the optimal receiver [Shih_Lee_IT03]

\[ C_{\text{opt}} = E \left[ \log \det \left( I_{N_r} + \frac{\rho}{N_t} \mathbf{HH}^H \right) \right] \]

\[ = e^{N_t/\rho} \sum_{i=0}^{N_r-1} \frac{\binom{N_r-1}{2N_r-N_t+i}^2}{2^{2i-2j}(N_r-N_t+i)!} \left( \frac{N_t}{\rho} \right)^{N_r-N_t+i} \sum_{r=0}^{i-j} \binom{2j-2i}{2j-2N_r-2N_t} \binom{2j-2i}{N_t/\rho} \]

Main Observations

- Two aspects to improve performances of the ZF receiver
  - working at high SNR regime
  - increasing the receive antennas
- The multiuser scheduling system creates an environment supporting both the above nutrients
- Main Idea: The inherent property of poor channel quality avoidance from multiuser scheduling provides a natural way to overcome this drawback
- How?

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Multiuser MIMO Downlink System

Feedback Mechanisms

- Vector Feedback
  - Spatially Greedy
  - Spatially Independent
- Scalar Feedback
  - Max-Min (SWNSF)
  - Max-Max

Spatially Greedy

Spatially Independent
Scalar Feedback: Max-Max

\[ \begin{align*}
\gamma_{1,1} & \\
\gamma_{2,1} & \\
\gamma_{3,1} & \\
\gamma_{2,2} & \\
\gamma_{3,2} & \\
\gamma_{3,3} & \\
\end{align*} \]

Scalar Feedback: Max-Min

\[ \begin{align*}
\gamma_{1,1} & \\
\gamma_{2,1} & \\
\gamma_{3,1} & \\
\gamma_{2,2} & \\
\gamma_{3,2} & \\
\gamma_{3,3} & \\
\end{align*} \]

Sum-rate analysis for ZF MIMO receivers after scheduling

- Sum-rate capacity subject to the spatially-independent scheduling

\[ C_{zf}^{\text{sl}} = N \int_0^\infty \log(1 + \gamma) f_{\gamma,n}(\gamma_n) d\gamma_n \]  
\[ = \frac{KN^2}{\rho} \sum_{i=0}^{K-1} \left( K - 1 \right) \left( -1 \right)^i h \left( \frac{(i + 1)N}{\rho} \right) \]

\[ h(x) \triangleq \int_0^\infty e^{-xt} \log(1 + t) dt = \frac{e^x E_1(x)}{x} \]

where \( E_1(x) \) is the exponential integer function of the first order.

Numerical Result

- Vector feedback scheduling
- Scalar feedback scheduling
- Spatially-independent scheduling with equal power allocation
- Spatially-independent scheduling with waterfilling power allocation

Number of users vs. System capacity
Output SNR distributions with Scheduling

Asymptotic Optimality of the ZF Receiver

In the multiuser MIMO system with SWNSF scheduling, for any finite number of antennas $N$, we have:

(i) At the low SNR regime
\[
\lim_{K \to \infty} \frac{C_{\text{zf}}}{C_{\text{opt}}} = 1, \quad \text{for small } \rho.
\]

(ii) At the high SNR regime
\[
\lim_{\rho \to \infty} \frac{C_{\text{zf}}}{C_{\text{opt}}} = 1, \quad \text{for any finite } K.
\]

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Observation

• We have shown that the multiuser diversity can be used to
  – not only improve system performance
  – but also potentially reduce implementation complexity
• Scheduling makes simple ZF-based MIMO receiver in the multi-user case approaches the performance of the optimal MIMO receiver.
Perspective on Future Research

- The amount of feedback plays a crucial role of “accelerator” in improving the downlink capacity of the multiuser MIMO system.

Conclusion

- ZF-receiver finds another life in the multi-user MIMO scheduling system.
- Is the top-down design method (scheduling scheme first then MIMO receiver design) feasible?
- Can it outperform the bottom-up design (MIMO receiver design first then scheduling scheme)?

Top-down or bottom-up design?

1. No simple answer!
2. The network-perspective (top-down) approach provides some exciting opportunities.

Reference

Thank You!

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