The Jointly Gaussian Approach to Iterative Detection in Multi-user – MIMO - ISI Channels

Xiaojun Yuan, Qinghua Guo and Li Ping
Overview

- Introduction
- Joint Gaussian (JG) approximation
- Iterative JG detection
- Performance analysis and optimization
- Numerical results
- Conclusions
Overview

- Introduction
- Joint Gaussian (JG) approximation
- Iterative JG detection
- Performance analysis and optimization
- Numerical results
- Conclusions
In an AWGN channel, we can assume that the signals are independent in time. This is approximately ensured by interleaving. (The signals are actually correlated by the coding constraint and this is the reason for iterative detection.) We can then use symbol-by-symbol Gaussian approximation.

\[ r(j) = \rho_k x_k(j) + \zeta_k(j) \]
In an ISI channel, the received signals are correlated in time. In this case, chip-by-chip Gaussian approximation may not be accurate.

Joint Gaussian approximation provides a better solution.
Consider a matrix system

\[ r = Hx + \eta \]

where

- \( r \) is the observation vector,
- \( x = \{x_j\} \) are binary inputs,
- \( H = [h_1, h_2, \ldots, h_j, \ldots] \) is a constant matrix and
- \( \eta \) is a vector of AWGN samples.
Linear System with Binary Input

\[ r = Hx + \eta \]

We assume that \( x \) contains binary entries. Such a system model is very common in practice, such as in ISI, MIMO and/or multi-user environments. The estimation in such a system is a very complicated problem, since it involves all possible combinations of binary inputs.

For example, we may evaluate all possibilities of all possible input combinations and select the one with maximum possibility. This is referred to as the maximum a posteriori probability (MAP) algorithm and it is very costly.

We will focus on a low cost alternative below.
A Philosophical View

We can use the following analogy to explain the difference between the MAP and iterative approach.

MAP is basically a socialist approach. We try to build a good society by finding an optimal solution. The problem is that the complexity involved is excessively high.

Iterative detection is a capitalist approach. It is like a two party political system in which “Republican” and “Democrats” take charge alternatively. This is definitely a sub-optimal approach. However, such a two-processor approach has realistic complexity.
Example 1: An ISI Channel

\[ r = Hx + \eta \]

\[
H = \begin{bmatrix}
  h_0 \\
  \vdots \\
  h_{L-1} \\
  \vdots \\
  h_0 \\
\end{bmatrix}
\]
Example 2: An MIMO-ISI Channel

\[ r = Hx + \eta \]

\[
H = \begin{bmatrix} H_{1,1} & H_{1,2} \\ H_{2,1} & H_{2,2} \end{bmatrix}, \quad H^{m,n} = \begin{bmatrix} h_{0,0}^{m,n} \\ \vdots \\ h_{L-1}^{m,n} \\ h_{L-1}^{m,n} \end{bmatrix}, \quad m = 1, 2, \quad n = 1, 2
\]

Here \( H^{m,n} \) is the ISI channel from antenna \( m \) to antenna \( n \).
Example 3: A Multi-user MIMO-ISI Channel

\[
r = Hx + \eta
\]

\[
H = \begin{bmatrix}
H^{1,1,1} & H^{1,2,1} & H^{1,1,2} & H^{1,2,2} \\
H^{2,1,1} & H^{2,2,1} & H^{2,1,2} & H^{2,2,2}
\end{bmatrix}
\]

for user 1

for user 2
Overview

• Introduction
• Joint Gaussian (JG) approximation
• Iterative JG detection
• Performance analysis and optimization
• Numerical results
• Conclusions
Joint Gaussian Approximation

Write

\[ r = Hx + \eta = \sum_m h_m x_m + \eta \]  
(1)

Focus on \( x_j \) and write

\[ r = h_j x_j + \xi_j \]  
(2)

where

\[ \xi_j \equiv \sum_{m \neq j} h_m x_m + \eta \]

We assume that \( \xi_j \) is joint Gaussian.

Then the detection in (1) is much simpler than that in (2), since (2) contains only one variable to be estimated. We can use a standard approach to estimating \( x_j \) in (2)
Co-variance Matrixes

The key problem here is $H$.

$$r = Hx + \eta = h_j x_j + \xi_j$$

Assume that the entries of $x$ are un-correlated after interleaving. Its co-variance can be written as $V = \nu I$. The co-variance of

$$\xi_j = \sum_{m \neq j} h_m x_m + \eta$$

is given by

$$R_j = R - \nu h_j h_j^T$$

where

$$R = HVH^T + \sigma^2 I$$

is the co-variance of $r$.

If $H$ is diagonal, the above reduces to the symbol-by-symbol Gaussian approximation.
Joint Gaussian Approach

Start from the JG approximation

\[ r = h_j x_j + \xi_j \]

where the co-variance of \( \xi_j \) is given by \( R_j = R - vh_jh_j^T \).

\[ \lambda_j \equiv \ln \frac{p(r \mid x_j = +1)}{p(r \mid x_j = -1)} \]

\[ = \ln \frac{\exp \left( -\frac{1}{2} (r - h_j - \mathbb{E}(\xi_j))^T R_j^{-1} (r - h_j - \mathbb{E}(\xi_j)) \right)}{\exp \left( -\frac{1}{2} (r + h_j - \mathbb{E}(\xi_j))^T R_j^{-1} (r + h_j - \mathbb{E}(\xi_j)) \right)} \]

\[ = 2h_j^T R_j^{-1} (r - H\mathbb{E}(x) + h_j \mathbb{E}(x_j)) \]

This can be written in a matrix form as

\[ \lambda = 2(I - VU)^{-1} [H^T R^{-1} (r - H\mathbb{E}(x)) + UE(x)] \]

where \( \lambda = [\lambda_0, \lambda_1, \lambda_2, \ldots ]^T \) and \( U \equiv (H^T R^{-1} H)_{\text{diag}} \).
A Summary on the JG Estimator

System model:
\[ r = Hx + \eta_j = h_j \chi_j + \xi_j \]

The JG estimator:
\[ \lambda = 2(I - VU)^{-1}[H^T R^{-1} (r - HE(x)) + UE(x)] \]

\[ R = \text{Cov}(r, r) = HVH^T + \sigma^2 I \]

\[ U \equiv (H^T R^{-1} H)_{\text{diag}} \]

\[ V = \text{Cov}(x, x) = \nu I \]
The JG Estimator

Repeat the JG estimator:

\[ \lambda = 2(I - VU)^{-1}[H^T R^{-1}(r - HE(x)) + UE(x)] \]

This estimator is actually equivalent to the LMMSE estimator Derived by Wang and Poor in the following paper:


However, the above approach is very concise and provides much insights into the problem.

We will consider efficient technique to compute the JG estimator later.
Iterative Detection

Key operation:

ESE: \[ \lambda = 2(I - VU)^{-1}[H^T R^{-1}(r - HE(x)) + UE(x)] \]
DEC: \[ \gamma \text{ (APP decoding)} \]

This is a vector form generalization of the symbol-by-symbol method.
Overview

- Introduction
- Joint Gaussian (JG) approximation
- Iterative JG detection
- Performance analysis and optimization
- Numerical results
- Conclusions
Iterative Detection

Key operation:

ESE: \[ \lambda = 2(I - VU)^{-1}[H^T R^{-1}(r - HE(x)) + UE(x)] \]
DEC: \[ \gamma \text{ (APP decoding)} \]
Overview

• Introduction
• Joint Gaussian (JG) approximation
• Iterative JG detection
• **Performance analysis and optimization**
• Numerical results
• Conclusions
Overview

- Introduction
- Joint Gaussian (JG) approximation
- Iterative JG detection
- Performance analysis and optimization
- Numerical results
- Conclusions
SNR Evolution for an SCM Detector

Evolution equation: \( SNR = U(I - VU)^{-1} \)

Evolution Trajectories

Proakis B channel [0.410 0.815 0.410]. Convolutional code (23, 35)₈ is used. Eb/No = 4 dB. J represents the transmission block length.
The Key Difficulty in Fading Channels

The EXIT chart technique is an efficient way to the analysis of iterative detectors in fixed channels based pre-simulated EXIT curves. However, a multi-user-MIMO-ISI channel typically involves fading. It is nearly impossible to produce EXIT curves for all possible channel conditions.
Solution

We can generate the channel EXIT curve using an on-line analytical method.

The curve for the decoder is still generated using simulation. It is invariant for channel conditions.

SNR-variance function is more convenient than mutual information
BER/FER Performance in Rayleigh Fading ISI Channels

Quasi-static Rayleigh fading ISI channel with different tap length $L$. Convolutional code $(23, 35)_8$ is used. Information length = 1024.
A Special Difficulty in MIMO Channels

In a SISO channel, all signals experience the same channel statistics. In this case, we can use a single $SNR$ value to approximate all the values in the (vector form) $SNR$.

In a MIMO channel, however, the signal transmitted from different antennas may experience very different channel conditions. In this case, we have to treat the inputs to the DEC carefully.
The FT and EMI Methods

- **Full-Table (FT) Method**: The DEC input sequence can be treated as samples with \( N \) (the total number of transmit antennas) different SNRs. An \( N \)-dimensional table is used to characterize the transfer function of the DEC. This method is very costly even for a moderate \( N \).

- **Equivalent Mutual Information (EMI) Method**: We replace the input LLR sequence \( \lambda \) (characterized by \( N \) SNR values) by another LLR sequence \( \lambda' \) with a single SNR and contains the same amount of mutual information (with respect to the transmitted signals) as \( \lambda \). Only one-dimensional tables are required in the evolution process.
BER/FER Performance in MIMO ISI Channels

Quasi-static Rayleigh fading $2 \times 2$ MIMO ISI channel with different $L$. Convolutional code $(7, 5)_8$ is used. Information length = 1024.
BER/FER Performance in MIMO ISI Channels

Quasi-static Rayleigh fading $4 \times 4$ MIMO ISI channel with different $L$. Convolutional code $(7, 5)_8$ is used. Information length = 1024.
BER/FER Performance in Multi-User MIMO ISI Channels

Quasi-static Rayleigh fading $2 \times 4$ MIMO ISI channel with $L = 4$. Convolutional code $(7, 5)_8$ is used. Information length $= 1024$. 
Overview

• Introduction
• Joint Gaussian (JG) approximation
• Iterative JG detection
• Performance analysis and optimization
• Numerical results
• Conclusions
Conclusions

JG is a very simple, but effective to model general linear systems driven by binary inputs.

An iterative JG approach has been developed for detection in multi-user-MIMO-ISI channels. Excellent performance has been demonstrated.

A analysis technique has been presented for the JG approach. The prediction agrees well with simulation results.

We will discuss efficient computation techniques for the JG approach in the next talk.